Adding Value through Risk Management in P&C

13rd September 2012, Milan – SIFA/FAC

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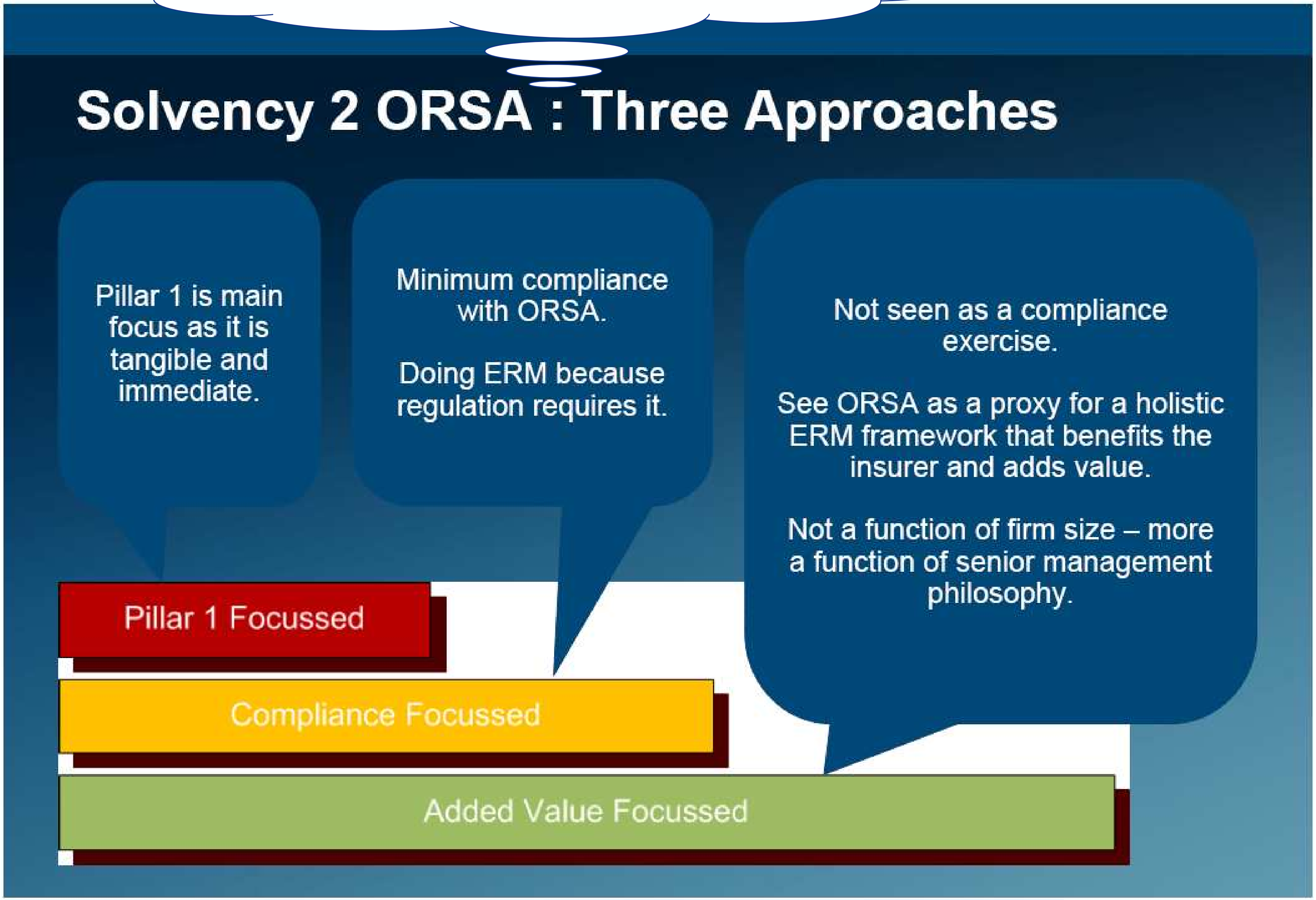
Extract from ECA2012*



(*) Elliot Varnell – ERM and the challenges for actuarial consultants
http://www.gcactuaries.org/documents/ECA2012_varnell.pdf

Agenda

10.45-12.45

1. Introduction
2. P&C Insurance Risks
   I. Reserve Risk
   II. Premium Risk
3. Risk Capital Aggregation and Allocation (Theoretical)

12.45-13.45

Lunch time ☺

13.45-16.00

4. Risk Capital Aggregation and Allocation (Practical)
5. Risk Based Pricing
6. Q&A Session

Contents

Laws and Regulations: Solvency II

- Solvency II is the **major European project on insurance legislation** for the next few years. Solvency II will lead to a completely different Supervisory System as well as enhanced use of Risk capital models and Risk management systems.
- Solvency II is the new proposed EU legislation which will govern the capital requirements of insurance companies.
- The current Solvency Framework, Solvency I, was introduced in the early 70's and defined capital requirements by specifying simple, factor-based solvency margins, which did not always reflect the true risks in a given portfolio of insurance business.
- Solvency II is an opportunity to improve insurance regulation and supervision, introducing a risk based economic approach.

Main goals of Solvency II

- Risk based Solvency II calculations: incentive for integrated risk management
- Convergence issues:
  - to Basel II in Europe
  - of supervisory approaches
- Market consistent valuation of assets and liabilities
- Group Supervision
- Increased transparency concerning supervisory practice and the business model of insurance companies
- Common market place – level playing field

Solvency II: a Three-Pillar Structure

Three Pillars Approach

PILLAR 1	PILLAR 2	PILLAR 3
Quantitative capital requirements	Qualitative requirements & supervision	Transparency
⇩	⇩	⇩
Quantitative Risk Management	Qualitative Risk Management	Disclosure

Three Pillars Approach is applied both at Company and at Group level

Pillar 1

- Assets and Liabilities Valuation
- Risk quantification
  - Minimum capital requirement (MCR)
  - Solvency capital requirement (SCR)
- Market value based own funds

Pillar 2

- Governance System
  - Organisational structure
  - Risk Management
  - Actuarial function
- Internal control system
- ORSA
- Use Test

Pillar 3

- Market information
  - Business overview and performances
  - Governance
  - Evaluation criteria
  - Risk and Capital Management
- Supervisory reporting

There are a lot of interdependencies between the different tasks within the different pillars

Purpose of the Risk Management

Putting all risks of a company on the scales ...

Risk exposure determines **required capital**

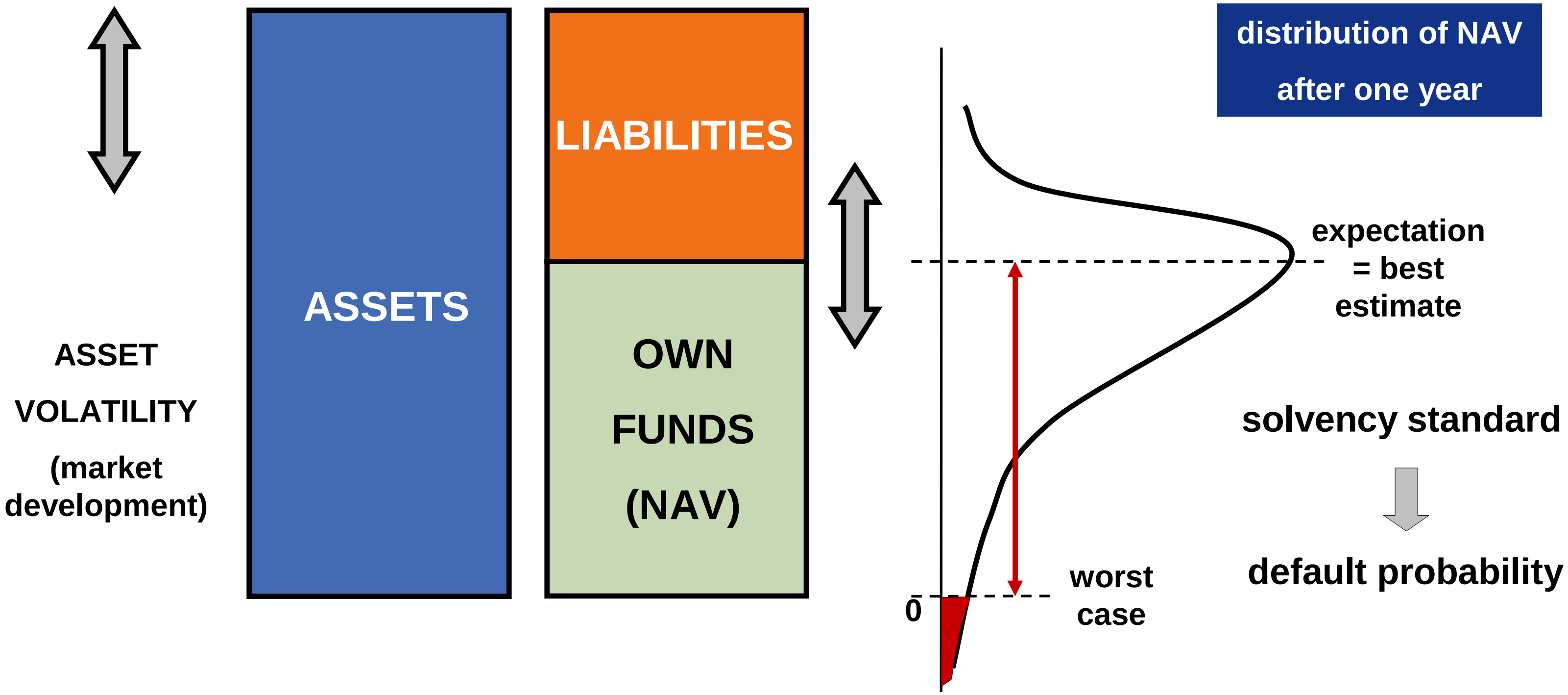
Risks taken have direct impact on the **value**!

Capital consumption comes at a **cost** (reduces the value)

… the idea is to correctly balance them, in order to create **value**!!

Risk Capital

SCR.1.9 The SCR (Solvency Capital Requirement) should correspond to the ***Value-at-Risk*** *of the* ***basic own funds*** *of an insurance undertaking subject to a confidence level of* ***99.5%*** *over a* ***one-year period***



So, everything that affects the own funds in the next 12 months should be considered as a **risk**

Risk Capital

Step 1: Assessment of nature, scale and complexity of risks

SCR.1.19 The insurer should assess the ***nature****,* ***scale*** *and* ***complexity*** *of the risks […]*

Step 2: Assessment of the model error

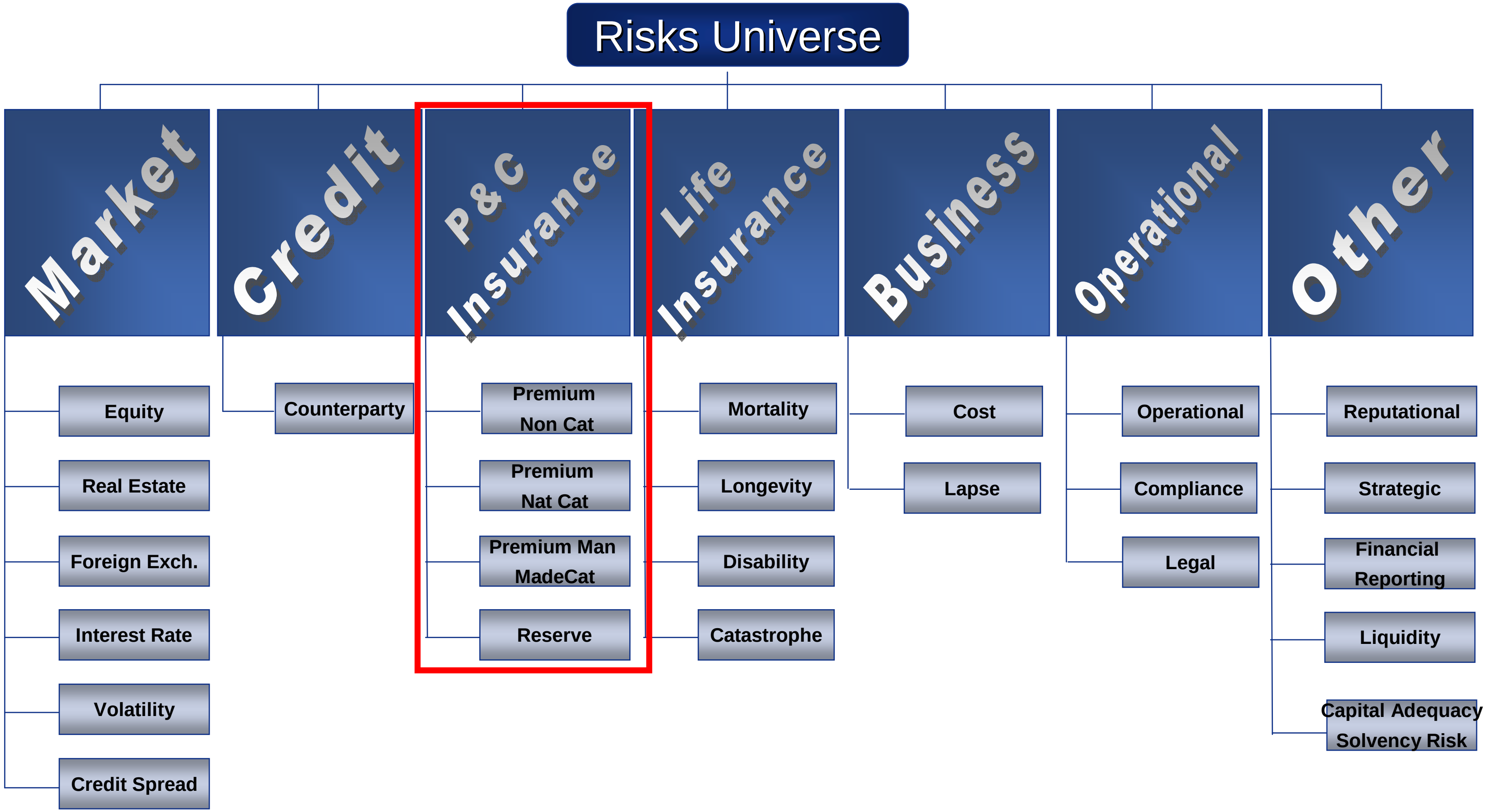
SCR.1.21 Where simplified approaches are used to calculate the SCR, this could introduce ***additional estimation uncertainty*** *(or* ***model error****) […]*

SCR.1.23 Undertaking ***are not required to quantify*** *the degree of* ***model error*** *in* ***quantitative terms*** *[…] Instead, it is sufficient* ***if there is reasonable assurance that the model error included in the simplifications is immaterial***

All the uncertainty – except of model error - should be considered in quantitative terms. This means that parameter and process error are in scope.

Risk Capital

Being the risk represented by the uncertainty of the future NAV development, this can be split into **several categories**, corresponding to the **events** giving place to the **possible NAV variations**



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Reserve Risk

SCR.9.11 Reserve risk results from ***fluctuations*** *in the* ***timing*** *and* ***amount*** *of* ***claim settlement***

Run off shock

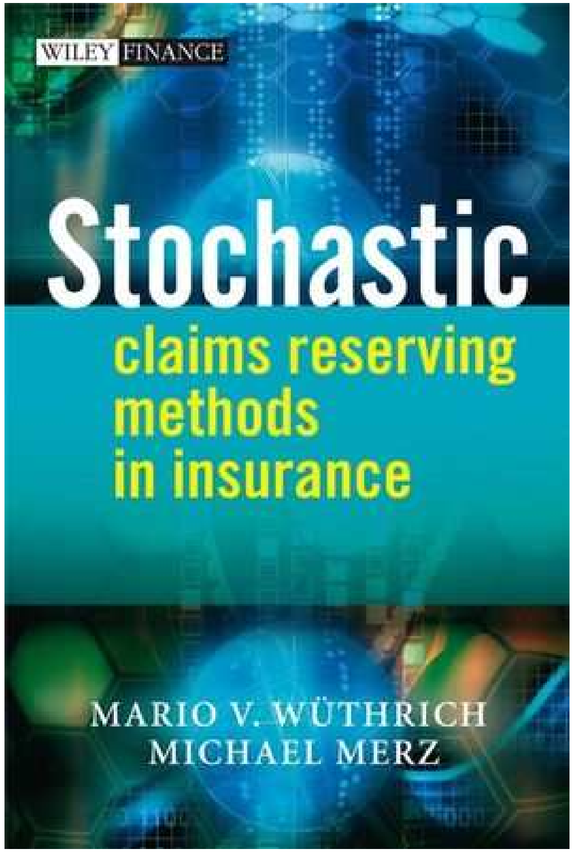
$$CDR_\infty = R_0 - \sum_{\text{future CY } t} P(t)$$

$$VAR\left(CDR_\infty\right) = VAR\left(\sum_{\text{future CY } t} P(t)\right)$$

In order words, it's like if we simulate the fact we are at the end of the reserve run-off and we observe how wrong we were at the instant of evaluation

Reserve Risk

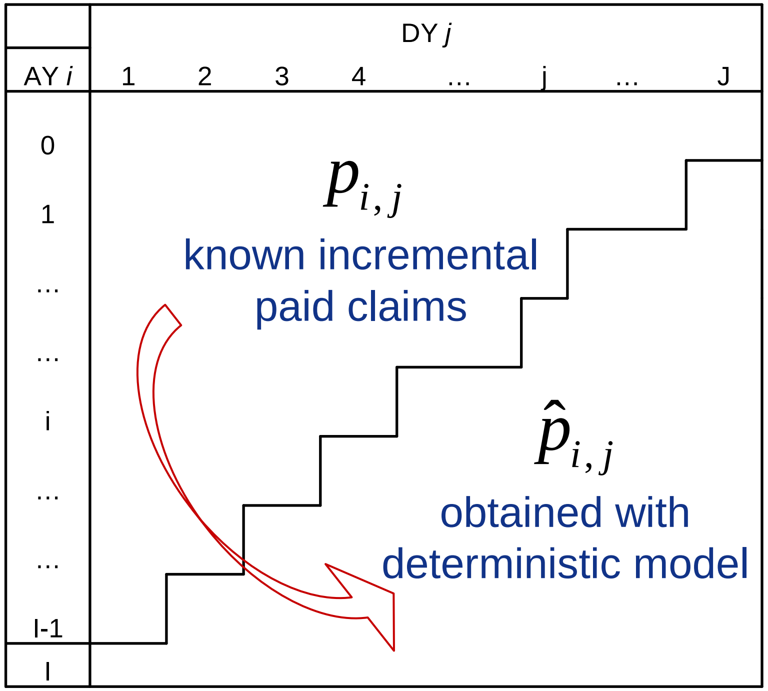
Tons of studies in actuarial literature regarding the Stochastic Loss Reserving

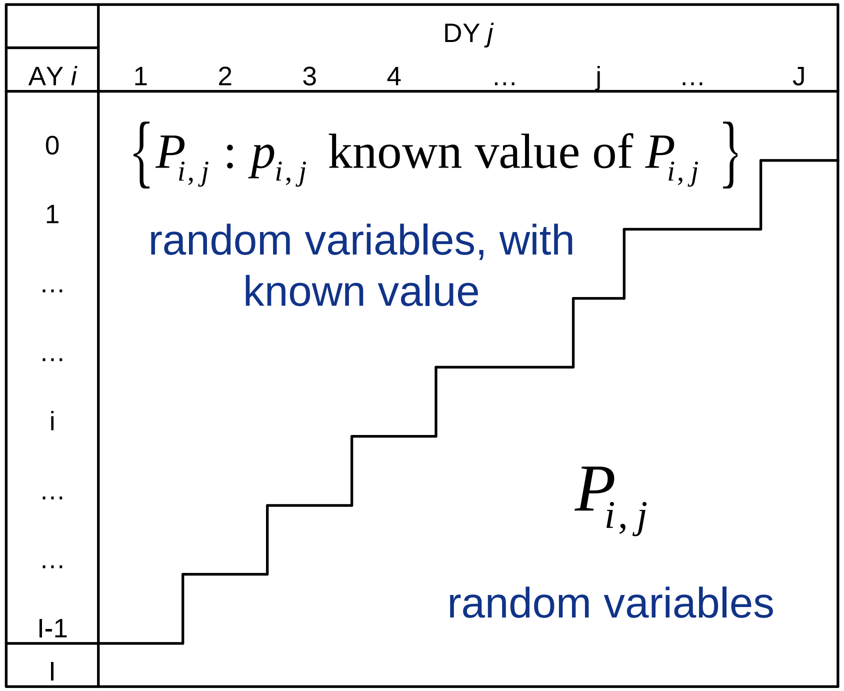


Reserve Risk – The underlying models

There is a “change of perspective” compared with the past

DETERMINISTIC MODELS → **STOCHASTIC MODELS**





Reserve Risk – The underlying models

In order to use stochastic model, you need to fix stochastic assumptions

Assumptions		Example
▪ PARAMETRIC	Give the parametric distribution family of $P_{i,j}$	GLM
▪ SEMIPARAMETRIC	Give only some assumptions on some moments	ODP / MACK

Usually the market is now considering **mainly** two stochastic models (ODP model & Mack model), and they are also generally accepted by Solvency II directive

But … what about the model error?

Reserve Risk – The underlying models

	ODP Model	Mack Model
Starting point	Calculate *Pearson Residuals* on **Incremental Paid(i,j)** Triangle	Calculate *Pearson Residuals* on **DFM(i,j)** Factors
Allowance for **Parameter Error**	Estimation of future payments via **bootstrapping**	Estimation of future payments via **recursive bootstrapping**
Allowance for **Process Error**	Adding process error using a Gamma (or other) distribution	Adding process error using a Gamma (or other) distribution

To do a correct bootstrap exercise, an analysis on residuals must be performed (they must be i.i.d) => **Residuals Analysis**

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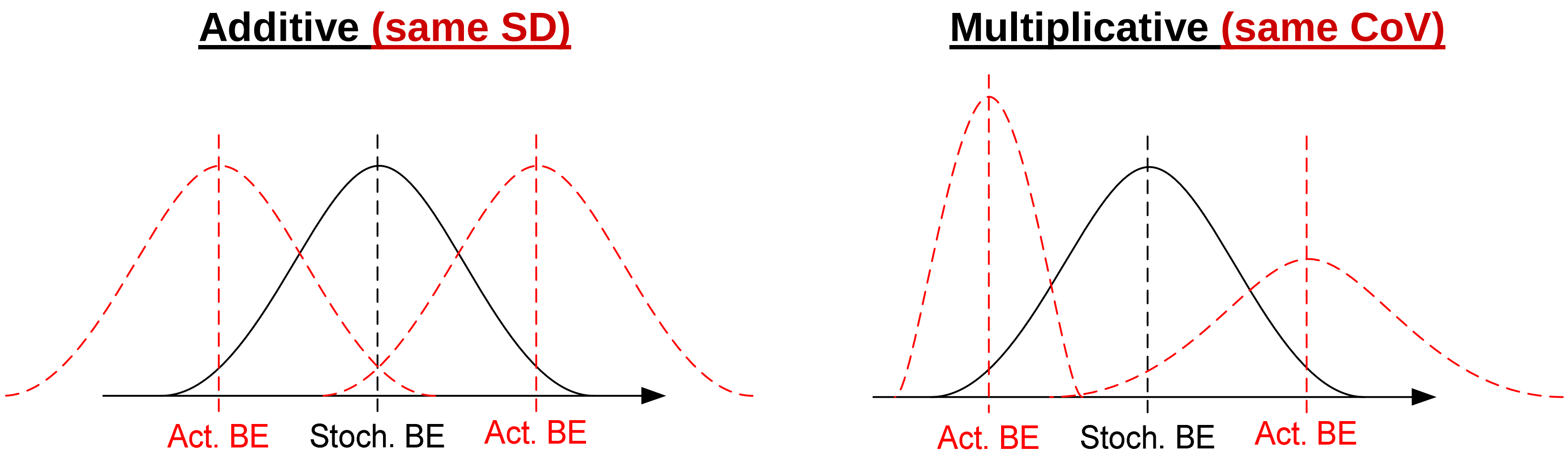
Reserve Risk - Shift to actuarial reserves

The implied expected value when assessing reserving uncertainty **does not necessarily correspond** to the best estimate reserve set by the Reserving Actuary (often based on different deterministic methods). However, we are still interested in the predictive distribution around the best estimate selected by the Reserving Actuary.

Two "types" of shift:



Generally, the common idea is to stay always on the safe side choosing a prudential approach (e.g. if ABE > BE then **multiplicative**, else **additive**)

Reserve Risk - Where are the problems?

Let's give a look again to the definition of SCR

SCR.1.9 The SCR (Solvency Capital Requirement) should correspond to the ***Value-at-Risk*** *of the* ***basic own funds*** *of an insurance undertaking subject to a confidence level of 99.5% over a* ***one-year period***

So, in 2008, the IAIS(*) published the following interpretation:

" […]

- ***Shock period****: the period over which a shock is applied to a risk;*
- ***Effect horizon****: the period over which the shock that is applied to a risk will impact the insurer*

In essence, at the end of the shock period, capital has to be sufficient so that assets cover the technical provision (…) ***re-determined at the end of the shock period****. The re-determination of the technical provisions would allow for the impact of the shock on the technical provisions over the full time horizon of the policy obligations"*

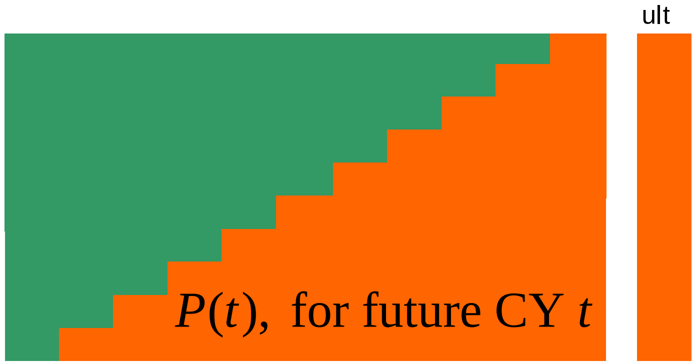
(*) International Association of International Supervisors

Guidance paper No. 2.1.1 on the structure of regulatory capital requirements (October 2008), Art. 55

http://www.iaisweb.org/view/element_href.cfm?src=1/5778.pdf

All the models seen until now consider a “shock” until the full reserve run-off (the so called “**Ultimate View**”)

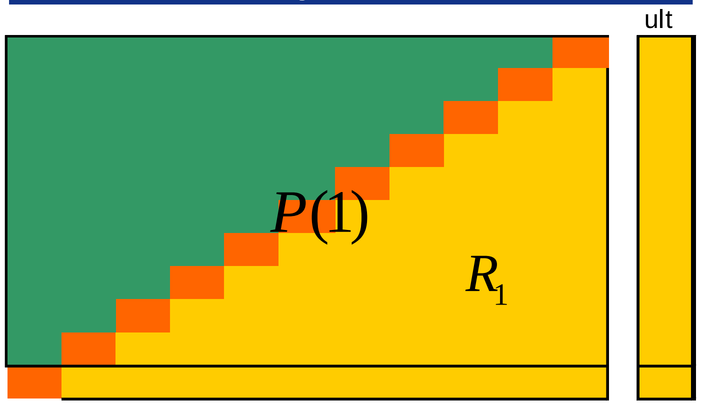
“Ultimate view”



$$CDR_{\infty} = R_0 - \sum_{\text{future CY } t} P(t)$$

$$VAR(CDR_{\infty}) = VAR\left(\sum_{\text{future CY } t} P(t) \right)$$

“1y view”



$$CDR_1 = R_0 - P(1) - R_1$$

$$VAR(CDR_1) = VAR(P(1) + R_1)$$

The “1-yr view” concept was born!!

Reserve Risk - The “1yr View”

From 2008, only **two** main studies have been performed on the topic

Merz, Wuthrich (June 2008) - Modelling the Claims Development Result for Solvency Purposes (ASTIN)

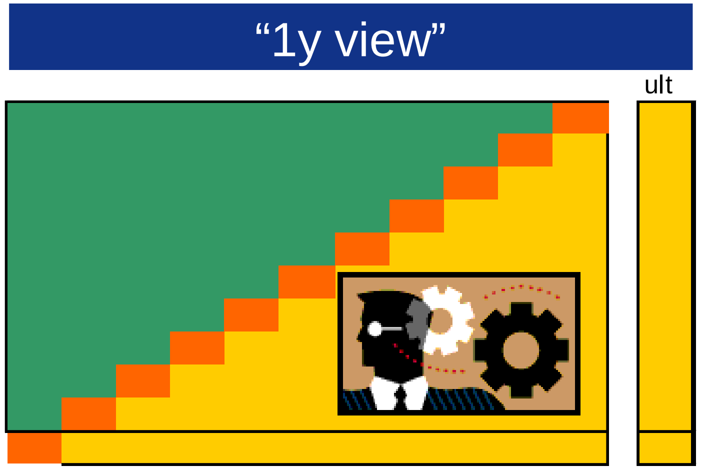
- Based on Mack (1993) assumptions + additional assumptions (martingale process)
- No tails considered
- Closed form for MSEP calculation (no information on the tails)
- Consider both a “perspective” and retrospective view

Starting from QIS5 it has been officially recognized for the calculation of the USP (Undertaking Specific Parameters), to be used thru a credibility approach with the market parameters

Reserve Risk - The "1yr View"

Ohlsson, Esbjorn, Lauzeningks (2008) - The one-year non-life insurance risk

- Gives only the general idea on how the one-year view should be evaluated (i.e. implementing the **re-reserving algorithm** – the so called "actuary in the box")
- If we consider as re-reserving algorithm only the CL, we get the previous Merz-Wuthrich approach



This approach is particular interesting for the internal model implementation; anyway, in these last years, not many studies in actuarial literature have been done: **there are still a lot of open issues to be deepen**

Reserve Risk - In a nutshell

The “**Ultimate View**” it has been commonly understood and accepted, but – practically – relies still too much to “CL-centered” models

…and **model error** should be kept under control (“the market does so” will be no more sufficient)!!

On the contrary, “**1yr View**” has still lot of interpretational issues and more studies have to be performed

Anyway, at the moment, the main “quantitative” issues are more or less solved and received a positive feedback from QIS5 exercise

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Premium Risk

*SCR 9.9. Premium risk results from fluctuations in the timing, frequency and severity of insured events. **Premium risk** relates to **policies to be written** (**including renewals**) during the period, and **to unexpired risks on existing contracts**. Premium risk includes the risk that **premium provisions** turn out to be **insufficient** to compensate claims or need to be increased.*

- It's an "atypical" risk, since it involves mainly P&L quantities (i.e. the future profit/losses) other than b/s figures (i.e. the Unearned Premium Reserve)

- Basically, it's a shock on the combined ratio for a given volume measure

$$RC = \left(CoR_{WC(99.5\%)} - CoR_{BE}\right) \cdot V_{(prem,lob)}$$

Premium Risk – Evaluating the “CoR shock”

$$\left(CoR_{WC(99.5\%)} - CoR_{BE}\right)$$

The **shock** is – more or less – quite easy to assess, typically based on historical CoR series:

- What about Premium Cycle uncertainty? It’s very difficult to assess its uncertainty
- Compared to Reserve Risk, are we considering an “ultimate” or a “1-yr view” ?
- And should we consider a correlation between losses and premiums “embedded” in the CoR?
- How to evaluate the impact of the Reinsurance structure?

Internal models often do a complete stochastic modeling (via a Montecarlo approach), useful also for other purposes (see Reinsurance Optimisation) … BUT …

Premium Risk – The volume measure

… the volume measure has been reviewed many times …

QIS5

$$V_{(prem,lob)} = \max\left(P_{lob}^{t,written}, P_{lob}^{t,earned}, P_{lob}^{t-1,written}\right) + P_{lob}^{PP}$$

P_{lob}^{PP} *Present value of net premiums of* ***existing contracts*** *which are expected to be earned after the following year for each LoBs*

L2 Draft

$$V_{(prem,s)} = \max(P_s; P_{(last,s)}) + FP_{(existing,s)} + FP_{(future,s)}$$

$FP_{(existing,s)}$ *Denotes the expected present value of premiums to be earned by the insurance undertaking in the segment s after the following 12 months for existing contracts*

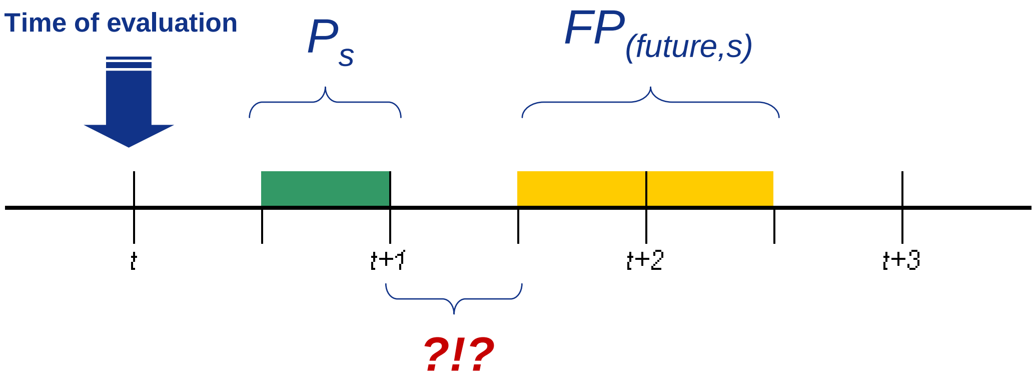
$FP_{(future,s)}$ *denotes the expected present value of premiums to be earned by the insurance undertaking in the segment s for contracts where the initial recognition date falls in the following 12 months but excluding the premiums to be earned during the 12 months* ***after the initial recognition date***

Premium Risk – The volume measure

… and it still have some issues ☺

Take for example a 2 years policy, with a single premium of 2.000€, that will incept in the middle of next year

The earnings of the contract as described the S2 volume measure are:



Reminder: $FP_{(future,s)}$ denotes the expected present value of premiums to be earned by the insurance undertaking in the segment s for contracts where the initial recognition date falls in the following 12 months but excluding the premiums to be earned during the 12 months ***after the initial recognition date (***it should say ***"after the next 12 months"!!!)***

Premium Risk - In a nutshell

Still not clear the perimeter of the Premium Risk, but it’s mainly related to earned premiums and UPR revaluation after one-year (why don’t call this latter “*Premium Reserve” Risk*?)

Other minor issues due to Premium Cycle interpretation and assumptions to be considered …

But the real problem regards **data quality and availability**: in the past data of future premiums have not been used in the balance sheet

Nat-Cat Risk – A quick overview

- "**Natural catastrophe**" ("**Nat Cat**") is a damaging event produced by nature elements followed by several single losses, involving a number of contracts (and then a number of contracting parties)
- The **extent** of a natural catastrophe depends on the force of the natural agent itself, but also on **man-made factors**, such as the quality of preventive measures adopted in the considered area, the technical building features, the maintenance level
- This risk includes:
  - **Earthquakes** (including seaquakes and tsunami)
  - **Flood**
  - **Hail**
  - **Hurricane**, **storm**, **avalanches**, **snow** and **freeze**

Usually, on a gross basis, the Nat-Cat risk "consume" a lot of capital …

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Risk Capital Aggregation

Basic setting

Suppose that the real-valued random variables $X_1,\ldots,X_n$ represent profits and losses for different assets in a portfolio (or P&L of different risks). Let X denote the total profit/loss of the portfolio, i.e.

$$X = X_1 + \ldots + X_n = \sum_{i=1}^{n} X_i$$

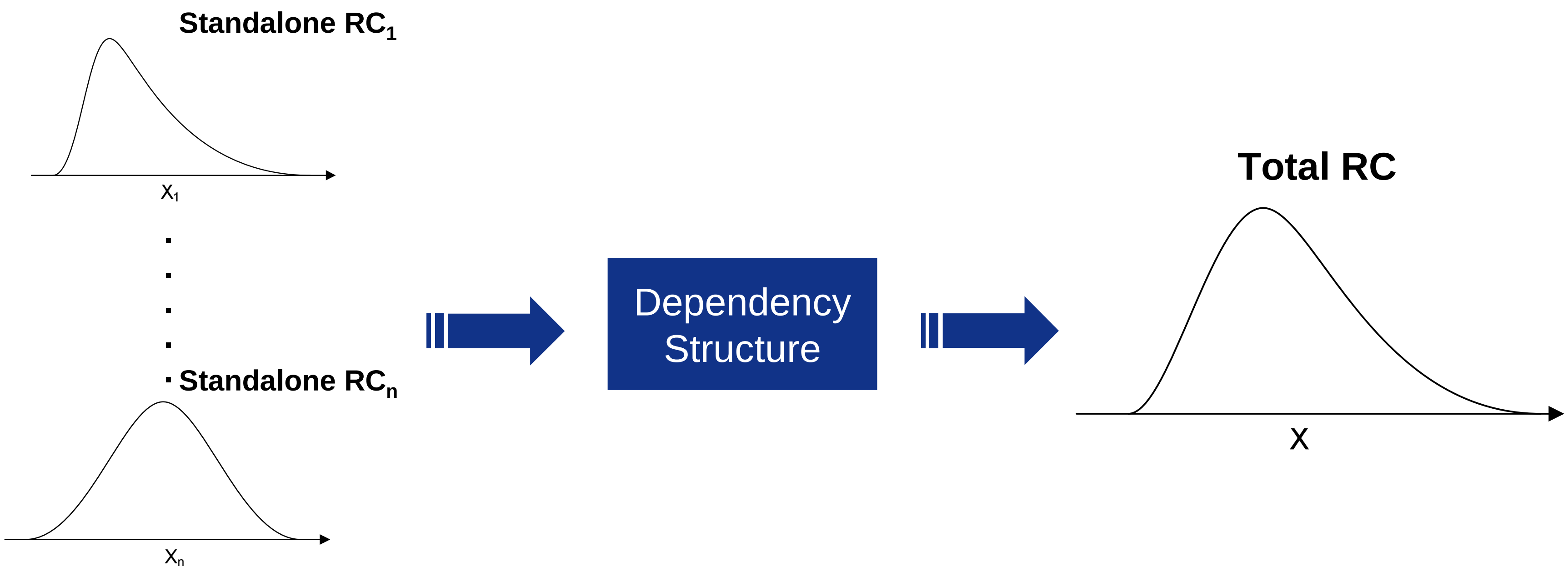
We can allow for some dynamics by introducing variables $u=(u_1,\ldots,u_n)$, thus representing with u_i the amount of money invested in asset i:

$$X(u) = X(u_1,\ldots,u_n) = \sum_{i=1}^{n} u_i X_i$$

Our aim is to model the total loss distribution as sum of different P&L sources, and for that we need to describe the dependence structure

Risk Capital Aggregation

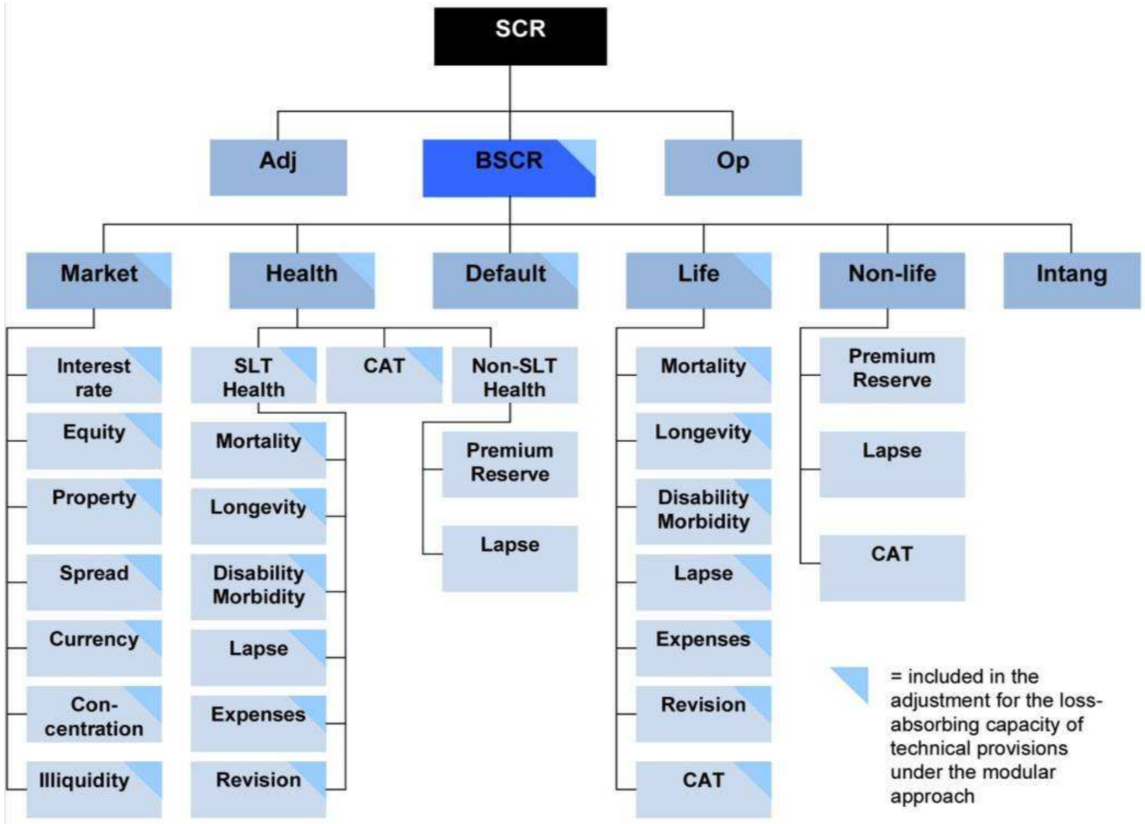
The idea of **Risk Aggregation**



We can calculate the standalone RC_i applying a **risk measure $\rho(.)$** to each $X_1,\dots,X_n$. The total risk capital is calculated applying the same risk measure to the distribution function of X.

Solvency 2 point of view

Solvency 2 standard formula capital requirement (SCR):



Inside some of the risk modules we also need to aggregate between LoBs

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Solvency 2 point of view

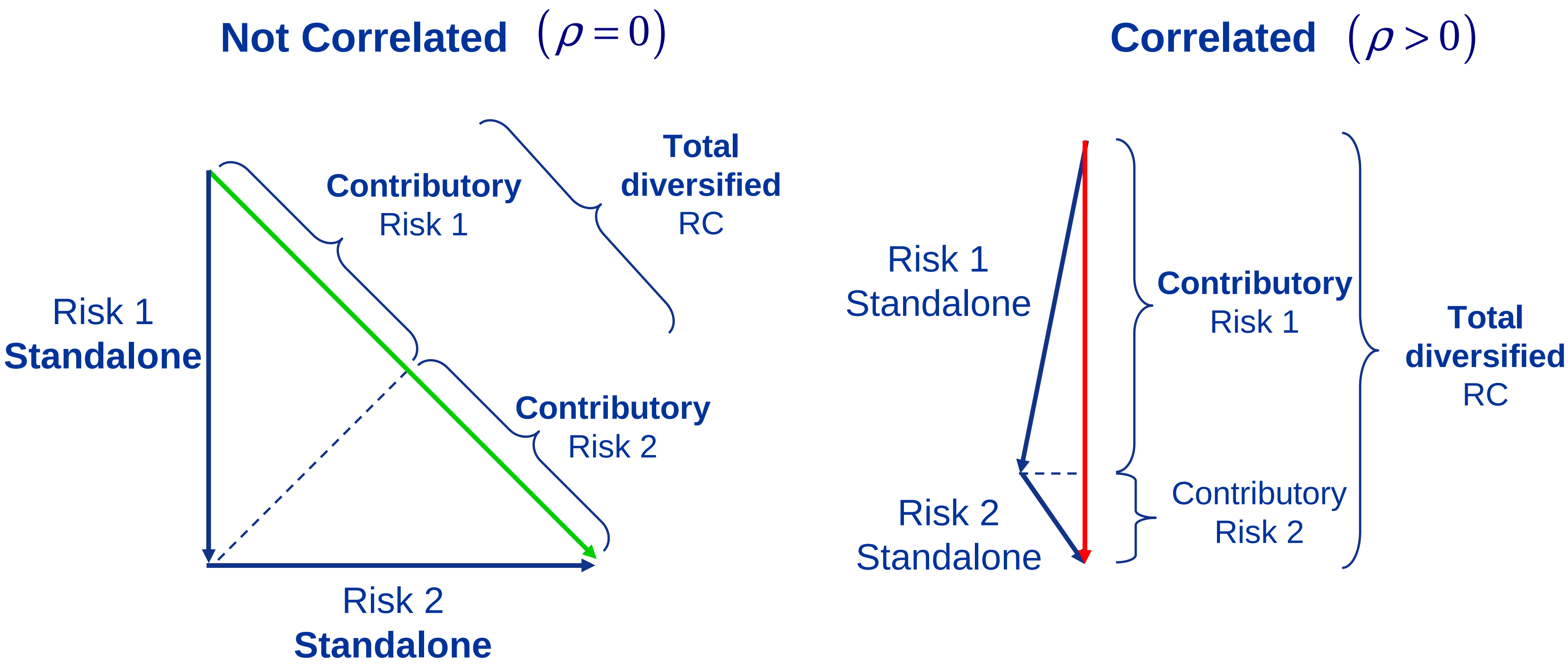
Solvency 2 formula for Risk Capital Aggregation

$$BSCR = \sqrt{\sum_{i,j} Corr_{i,j} \cdot SCR_i \cdot SCR_j}$$

Note: the same logic applies for the aggregation of sub-modules into the main risk modules, and for the aggregation of LoBs inside sub-modules

The proposed aggregation is entirely based on linear correlations…
…is this ok?

The concept of linear aggregation



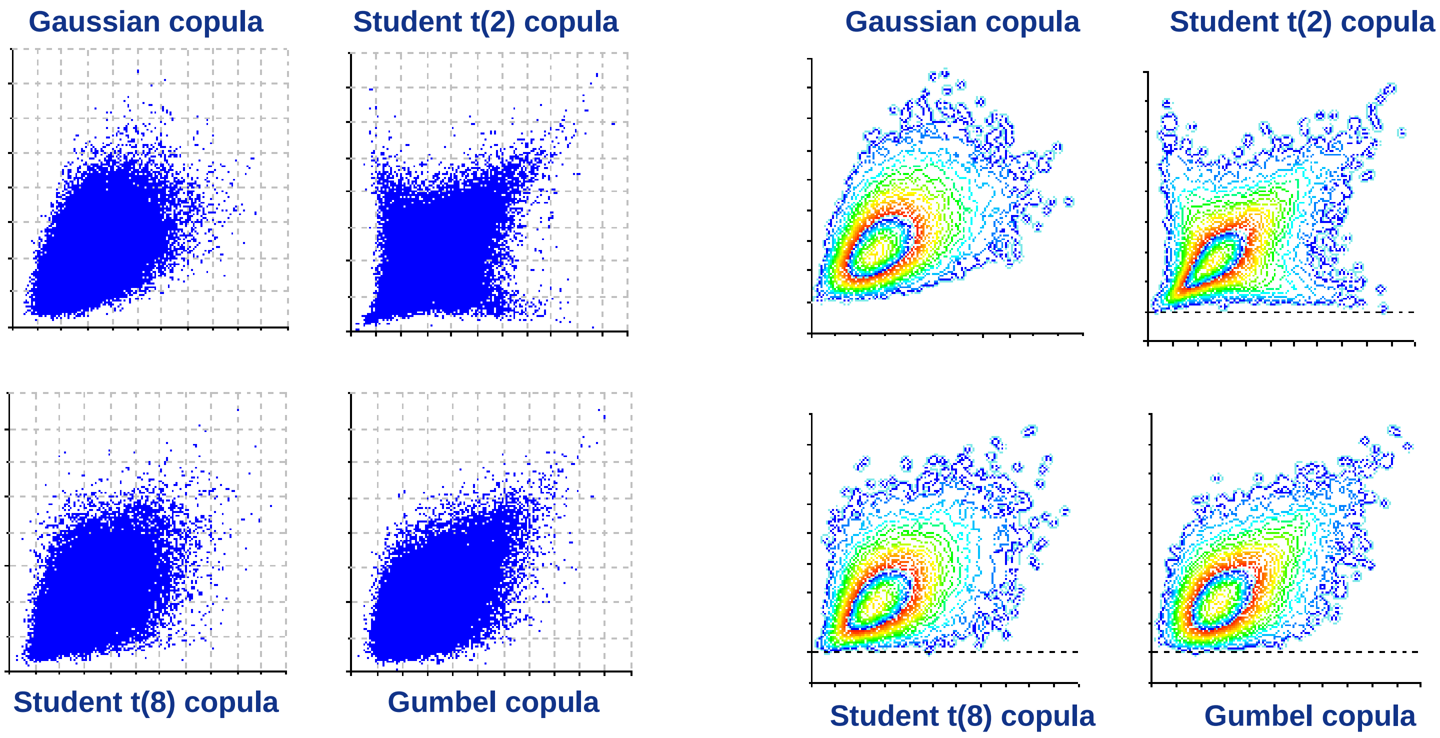
- The **length** of the vectors is the value of the **RC standalone**
- The **angle** represents the **linear correlation** (90°= not correlated, 180°= full correlated)

Some of the problems of using linear correlations…

1. The possible values of the linear correlation coefficient ρ could be restricted to a sub interval of [-1,1]
2. The marginal distributions needs to have finite variance
3. **The marginal distributions and the correlation coefficient are not enough to uniquely represent the joint distribution.**
4. If we take two univariate distributions F1, F2 and a correlation coefficient ρ from [-1,1], it is not always possible to build a joint distribution function F with marginals F1, F2 and correlation coefficient ρ.
5. The linear correlation coefficient ρ depends not only from the copula, but also from the marginals (other correlation indexes are more appropriate, e.g. the spearman and kendall rank correlation indexes)

“The devil is in the tails”

(Donnelly, Embrechts, 2010, Astin bulletin)



Starting with the same marginals and the same correlation coefficient, we can obtain very different joint distributions, especially in the tails…

What really is a copula?

Copula definition: a *d*-dimensional copula $C(u)=C(u_1,\ldots,u_d)$ is a cumulative distribution function over $[0,1]^d$ with uniform marginals in $[0,1]$, i.e. C is such as $C : [0,1]^d \rightarrow [0,1]$

Sklar's Theorem: given a joint cumulative distribution function F with marginals $F_1,\ldots,F_d$, there always exists a copula C that links the given marginals with the joint distribution F. Besides, if the marginals are all continuous the copula is unique.

for the "proper" definition and theorem see, for example: McNeil, Frey, Embrechts (2005) "Quantitative risk management"

> The copula is basically a function that links the marginals to the joint distribution, and includes all the information about the dependence structure

Some well-known copulas…

Gaussian copula

$$C_P^{Ga}(\mathbf{u}) = \Pr[\Phi(X_1) \le u_1, \ldots, \Phi(X_d) \le u_d] = \mathbf{\Phi}_P(\Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_d))$$

- The Gaussian copula implies asymptotic independence in the tails

Student-t copula

$$C_{P,\nu}^{t}(u_1, \ldots, u_d) = T_{P,\nu}(t_\nu^{-1}(u_1), \ldots, t_\nu^{-1}(u_d))$$

- This copula needs one more parameter (d.o.f.), but it is now possible to introduce asymptotic tail dependence

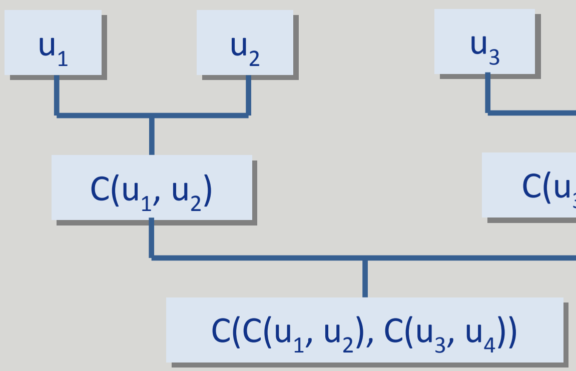
Some well-known copulas…

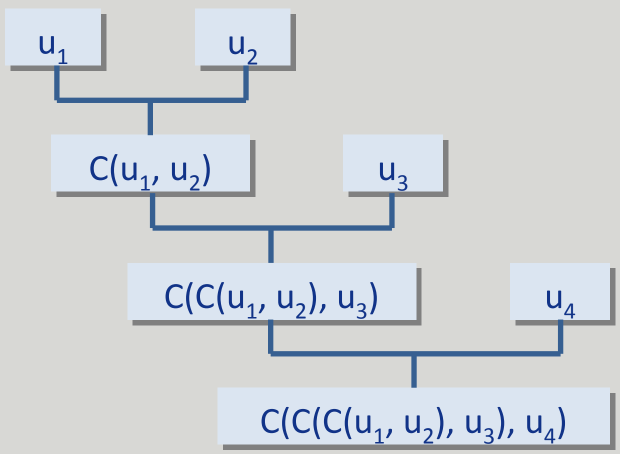
Archimedean copulas (Gumbel, Clayton, Frank, etc)

$$C(u_1,\ldots,u_d)=\phi^{-1}(\phi(u_1)+\ldots+\phi(u_d))$$

- Those copulas are heavyer on the tails, but they are also a little restrictive since the dependence structure is described by only one parameter

Hierarchical Archimedean copulas





- Very difficult to parameterize (nesting/estimation) and to simulate from

4 - Applying Risk Measures to Allocation Methods

Alternative approaches	
1. A simpler model (e.g. **linear correlations**) is not detailed enough	☒
2. **Directly estimating the joint distribution** is much more difficult	☒
3. Both **simulating directly from the joint distribution** or **calculating analytically the convolution** is a very difficult and computationally demanding	☒

PROs of the Copula	
1. We can separate the analysis of the marginals from the analysis of the dependency structure	✓
2. To estimate the copula parameters we can use the usual estimation techniques	✓
3. For the simulation we can use a simple *reordering algorithm*	✓

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Introduction

The idea of **Capital Reallocation**

Once we have the total risk capital (that takes into account dependencies and diversification effect), we want to re-allocate this figure to the single risks.

One of the main aims of Capital Reallocation is to compute the Cost of Capital relative to single risks/LoBs, in order to **determine risk adjusted profitability of single segments.**

Risk Measures

Definition of **Risk Measure**

In actuarial literature, a risk measure is defined as a mapping ρ from a set L of real-valued random variables defined on a probability space (Ω, ζ, λ) to the real line R:

$$\rho : L \to \mathbb{R}$$

$$\text{i.e.} \quad \rho : X \in L \to \rho(X)$$

Usually, if X represents a random return, from an economic point of view we can see $\rho(X)$ as the (positive) **amount of capital** set aside in order to make X an acceptable risk (*)

(*) Generally the concept of setting aside a Risk Capital to hedge the risks is a direct consequence of the "translation invariance" property (see slide 9), i.e. setting aside a Risk Capital that equals $\rho(X)$ makes the total risk measure go to zero.

Risk Measures

Definition of **Coherent Risk Measure**

A risk measure ρ is called ***coherent*** if it satisfies the following properties *([1] Artz et. al. (1999))*:

$$(i)\quad \text{Translation Invariance: } \rho(X+\alpha)=\rho(X)-\alpha,\ \forall X\in L, \alpha\in\mathbb{R}$$

$$(ii)\quad \text{Subadditivity: } \rho(X+Y)\le\rho(X)+\rho(Y),\ \forall X,Y\in L$$

$$(iii)\quad \text{Positive Homogeneity: } \rho(\beta X)=\beta\rho(X),\ \forall X\in L, \beta\ge 0$$

$$(iv)\quad \text{Monotonicity: } \rho(X)\le\rho(Y),\ \forall X,Y\in L \text{ with } X\ge Y\ \lambda\text{-a.s.}$$

These properties are desirable because they all have an interpretation that is logical from an economic point of view (e.g. without the Subadditivity, the risk measure can show anti-diversification)

Risk Measures

Some well-known Risk Measures:

$$(i)\quad \text{Variance:}\ \rho_{\text{var}}(X) = \text{var}(X)$$

$$(ii)\quad \text{Standard deviation:}\ \rho_{sd}(X) = \sigma(X) = \sqrt{\text{var}(X)}$$

$$(iii)\quad \text{Value-at-Risk (VaR):}\ \rho_{VaR(\alpha)}(X) = -VaR_{(\alpha)}(X)$$

$$(iv)\quad \text{TVaR}^{1}\text{:}\ \rho_{TVaR(\alpha)}(X) = -E(X \mid X \le VaR_{(\alpha)}(X))$$

Please note that in this case α is a ‘left-side’ number, e.g. 0.5% for Solvency II

Among these, only the TVaR is a **coherent** risk measures.

1. In our assumptions the underlying distribution is continuous, therefore the TVaR (also called Tail Conditional Expectation) is the same as the Expected Shortfall (*[2] Acerbi, Tasche (2002)*).

Allocation Methods

Definition of Allocation Method

We denote by M the set of risk measures. A capital allocation method Φ is a mapping

$$\Phi : M \times L^n \to \mathbb{R}^n, \quad (\rho, X_1, ..., X_n) \to \begin{pmatrix} \Phi_1(\rho, X_1, ..., X_n) \\ ... \\ \Phi_n(\rho, X_1, ..., X_n) \end{pmatrix}$$

Therefore, an allocation method is defined once we choose a **particular form for the functional Φ**.

Allocation Methods

From now on we will use the following notation:

- $\rho(X)$ for the risk capital allocated to the total portfolio
- $\rho(X_i)$ for the standalone risk capital allocated to the single risk
- $\rho(X_i|X)$ for the contributory risk capital reallocated to the single risk

Besides, we can briefly introduce the concept of Return On Risk Adjusted Capital (**RORAC**):

- related to the total (*): $RORAC(X) = \frac{E(X)}{RC_{TOT}} = \frac{\sum_{i=1}^{n} \mu_i}{\rho(X)}$
- related to the single contribution (*): $RORAC(X_i) = \frac{E(X_i)}{RC_i} = \frac{\mu_i}{\rho(X_i \mid X)}$

(*) Please note that in this notation $\rho(X)$ = RC, e.g. $\rho(X) = [-VaR_{\alpha}(X)] - E(X)$

Allocation Methods

“Definition” of **Coherent Allocation Method**, and economic reasons on why we should look for a coherent allocation method

(i) Full allocation property: $\sum_{i=1}^{n} \rho(X_i \mid X) = \rho(X).$

comment: it is obvious that we need the single contributory risk capitals to add up to the total risk capital. By the way, this condition is easy to obtain just by putting some constraints on the coefficients Φ_i

(ii) Diversifying allocation property: $\rho(X_i \mid X) \leq \rho(X_i),\ i = 1,\ldots,n$

comment: we clearly want to take diversification into account when reallocating risk capital to the single risks/Lobs

Allocation Methods

(iii) RORAC compatibility property:

$$RORAC(X_i \mid X) > RORAC(X) \;\Rightarrow RORAC(X + hX_i) > RORAC(X)$$

for all $h > 0,\ i = 1,\dots,n$

comment: without this property the reallocation process results could be economically odd

These properties are desirable because they all have an interpretation that is logical from an economic point of view. It is reasonable to look for allocation methods that verify them.

*OSS: Generally speaking **there is no reason to impose non-negative coefficients** (e.g. to allow for hedging effects) (*)*

(*) Please note that there is no allocation method that always verify the non-negativity, but we may need it if we want to use RAPM-type quotients (return/allocated capital); see *[4] Denault (2001)* for an insight on the non-negativity property for reallocation methods

Allocation Methods

Examples of Allocation Principles

(i) Proportional: $\Phi_i^{P,\rho} = \rho(X_i) \Big/ \sum_{j=1}^{n} \rho(X_j)$

(ii) Marginal Approach (or Merton & Perold): $\Phi_i^{MP,\rho} = \dfrac{\rho(X) - \rho(X - X_i)}{\sum_{j=1}^{n} \left[\rho(X) - \rho(X - X_j) \right]}$

(iii) Euler Allocation (or Myers & Read): $\Phi_i^{Eu,\rho} = \dfrac{\partial \rho(X)}{\partial u_i} \Big/ \sum_{j=1}^{n} \dfrac{\partial \rho(X)}{\partial u_j}$

It is shown in the actuarial literature that only the Euler Allocation applied to a coherent risk measure can be a coherent allocation method

Applying Risk Measures to Allocation Methods

How to build a **Capital Reallocation Approach**

As we've seen, we can write:

$$RC_i = \Phi_i \cdot RC_{TOT} = \frac{\rho^a(X_i \mid X)}{\sum_{i=1}^{n} \rho^a(X_i \mid X)} \cdot \rho^{TOT}(X)$$

Therefore, to fully describe the allocation approach we need to define:

- Φ is the function that describes the **allocation principle**
- ρ^a is the risk measure used inside the functional Φ
- ρ^{TOT} is the risk measure used to obtain the total risk capital

In the actuarial literature ρ^a and ρ^{TOT} are always the same, since this is required to have some good properties verified. In the practice they are often different.

Applying Risk Measures to Allocation Methods

"Standalone Approach"

Allocation principle (Φ)	Risk measure for re-allocation (ρ^a)	Risk measure for total RC (ρ^{TOT})
Proportional	Usually VaR or TVaR	Usually VaR or TVaR

PRO

· Very easy to calculate

CONS

· Does not take into account intra-risks diversification benefit, and **this is true for every risk measure we choose**

Applying Risk Measures to Allocation Methods

"Marginal Approach"

Allocation principle (Φ)	Risk measure for re-allocation (ρ^a)	Risk measure for total RC (ρ^{TOT})
Marginal (Merton & Perold)	Usually VaR or TVaR	Usually VaR or TVaR

PRO

· "What if" point of view

CONS

· Computational demanding, we need the joint distribution of (X - X_i) for every i

· For any risk measure, **it is not a coherent allocation method** (*[11] Tasche (2007)*)

This method may be useful to analyse the impact of adding/removing one entire LoB, but it is not suited to reallocate the Cost of Capital

Applying Risk Measures to Allocation Methods

"Covariance Approach"

Allocation principle (Φ)	Risk measure for re-allocation (ρ^a)	Risk measure for total RC (ρ^{TOT})
Euler (Myers & Read)	**Standard deviation**	Usually VaR or TVaR

This could be a coherent allocation method, but only if we use the **standard deviation** for **both** ρ^a and ρ^{TOT}. Please note that the standard deviation could be a coherent risk measure under more restrictive assumptions.

If we want to use the VaR or TVaR for computing the total Risk Capital, then this is not a coherent allocation method (*[10] Tasche (2004)*, p.14)

Applying Risk Measures to Allocation Methods

"Decomp VaR Approach"

Allocation principle (Φ)	Risk measure for re-allocation (ρ^a)	Risk measure for total RC (ρ^{TOT})
Euler (Myers & Read)	VaR	Usually VaR or TVaR

PRO

- Very easy to calculate

CONS

- Very unstable, and not a reliable measure of risk contributions

Applying Risk Measures to Allocation Methods

"VaR HD"

Allocation principle (Φ)	Risk measure for re-allocation (ρ^a)	Risk measure for total RC (ρ^{TOT})
Euler (Myers & Read)	VaR HD	Usually VaR (HD) or TVaR

PRO

· "Decent" representation of risk contributions

CONS

· It is not a coherent capital allocation method, therefore it can sometimes give odds results

Applying Risk Measures to Allocation Methods

"Contribution to TVaR Approach"

Allocation principle (Φ)	Risk measure for re-allocation (ρ^a)	Risk measure for total RC (ρ^{TOT})
Euler (Myers & Read)	**TVaR**	Usually VaR or **TVaR**

It's possible to prove that if we use the **TVaR to measure the total risk capital and to reallocate, then** we have a **coherent allocation method** with a **coherent risk measure** (*TVaR*) - (*[10] Tasche (2004)* and *[11] Tasche (2007)*)

Applying Risk Measures to Allocation Methods

In fact, following this proposal we obtain:

$$RC_i = \Phi_i \cdot RC_{TOT} = \frac{TVaR_\alpha(X_i \mid X)}{\sum_{i=1}^{n} TVaR_\alpha(X_i \mid X)} \cdot TVaR_\alpha(X)$$

$$= \frac{E[X_i \mid X \geq VaR_\alpha(X)]}{\sum_{i=1}^{n} E[X_i \mid X \geq VaR_\alpha(X)]} \cdot E[X \mid X \geq VaR_\alpha(X)] = E[X_i \mid X \geq VaR_\alpha(X)]$$

$$RC_i = E[X_i \mid X \geq VaR_\alpha(X)]$$

Applying Risk Measures to Allocation Methods

PROs	
1. We use a **coherent risk measure** to calculate the total risk capital	✓
2. We use a **coherent capital allocation** method	✓
3. It is **very easy to calculate** the single contributions (see next)	✓
4. **it uses all the information** from both the total distribution and the marginal distributions	✓
5. It can be used for different economic purposes just by changing the α (i.e. if we want a risk management approach we can focus on the tails with α=0.5%, if we want a more "expected" approach we can set α=50%)	✓

CONs	
It doesn't work for non-linear risks (e.g. Market Risks)	☒
The TVaR requires at least 10'000 sims at the 99% confidence level to give stable results (*[13] Yamai, Yoshiba (2007)*)	☒

Contents

Reordering algorithm

Illustration based on a simple problem: $(X, Y) \sim C(F_X, F_Y)$

1. Fix $n \in \mathbb{N}$
2. Simulate independently
   - $X_i \sim F_X$
   - $Y_i \sim F_Y$
   - $U_i = (U_i^1, U_i^2) \sim C$

   for $i = 1, \ldots, n$
3. Construct “samples” of (X, Y) by merging the order statistics $X_{(i)}$ and $Y_{(i)}$ according to the observed joint ranks in the copula sample.

Extract from *Philipp Arbenz (2012) – High dimensional risk aggregation: A Hierarchical approach with Copulas,* www.math.ethz.ch/~arbenz/

Reordering algorithm: Sampling marginals and Copula

Let $n = 4$

1. Sample i.i.d. $X_i \sim F_X$,
2. Sample i.i.d. $Y_i \sim F_Y$, independent of the X_i
3. Sample i.i.d. $U_i \sim C$, $U_i \in [0,1]^2$, independent of the X_i and Y_i

$X_i \sim F_X$		$Y_i \sim F_Y$		$U_i \sim C$	
sample	rank	sample	rank	sample	rank
3.1	2	67.9	4	(0.4, 0.7)	(2,3)
6.3	4	22.8	2	(0.5, 0.9)	(3,4)
1.4	1	12.2	1	(0.1, 0.3)	(1,1)
5.9	3	43.7	3	(0.7, 0.4)	(4,2)

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$X_i \sim F_X$		$Y_i \sim F_Y$		$U_i \sim C$	
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3. Sample i.i.d. $U_i \sim C$, $U_i \in [0,1]^2$, independent of the X_i and Y_i

$X_i \sim F_X$ sample	$X_i \sim F_X$ rank	$Y_i \sim F_Y$ sample	$Y_i \sim F_Y$ rank	$U_i \sim C$ sample	$U_i \sim C$ rank
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3. Sample i.i.d. $U_i \sim C$, $U_i \in [0,1]^2$, independent of the X_i and Y_i

$X_i \sim F_X$ sample	rank	$Y_i \sim F_Y$ sample	rank	$U_i \sim C$ sample	rank
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Samples of (X, Y):

(,)
(,)
(,)
(,)

Extract from *Philipp Arbenz (2012) – High dimensional risk aggregation: A Hierarchical approach with Copulas,* www.math.ethz.ch/~arbenz/

Allianz

Reordering algorithm: Reordering

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(3.1,)
(,)
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(,)

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Reordering algorithm: Reordering

$X_i \sim F_X$ sample	$X_i \sim F_X$ rank	$Y_i \sim F_Y$ sample	$Y_i \sim F_Y$ rank	$U_i \sim C$ sample	$U_i \sim C$ rank
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(3.1,)
(,)
(,)
(,)

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Allianz

Reordering algorithm: Reordering

$X_i \sim F_X$ sample	rank
3.1	2
6.3	4
1.4	1
5.9	3

$Y_i \sim F_Y$ sample	rank
67.9	4
22.8	2
12.2	1
43.7	3

$U_i \sim C$ sample	rank
(0.4, 0.7)	(2,3)
(0.5, 0.9)	(3,4)
(0.1, 0.3)	(1,1)
(0.7, 0.4)	(4,2)

Samples of (X, Y):

(3.1, 43.7)
(,)
(,)
(,)

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5.9	3	43.7	3	(0.7, 0.4)	(4,2)

Samples of (X, Y):

(3.1, 43.7)
(5.9, 67.9)
(1.4, 12.2)
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Samples of (X,Y): → samples of $X+Y$

(X,Y)	$X+Y$
(3.1, 43.7)	3.1 + 43.7
(5.9, 67.9)	5.9 + 67.9
(1.4, 12.2)	1.4 + 12.2
(6.3, 22.8)	6.3 + 22.8

Extract from *Philipp Arbenz (2012) – High dimensional risk aggregation: A Hierarchical approach with Copulas,* www.math.ethz.ch/~arbenz/

Reordering algorithm: Reordering

$X_i \sim F_X$		$Y_i \sim F_Y$		$U_i \sim C$	
sample	rank	sample	rank	sample	rank
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1.4	1	12.2	1	(0.1, 0.3)	(1,1)
5.9	3	43.7	3	(0.7, 0.4)	(4,2)

Samples of (X, Y):

(3.1, 43.7)
(5.9, 67.9)
(1.4, 12.2)
(6.3, 22.8)

samples of $X + Y$

3.1 + 43.7 = 46.8
5.9 + 67.9 = 73.8
1.4 + 12.2 = 13.6
6.3 + 22.8 = 29.1

Extract from *Philipp Arbenz (2012) – High dimensional risk aggregation: A Hierarchical approach with Copulas,* www.math.ethz.ch/~arbenz/

Reallocation in Practice

Initial assumptions:

- We know (or have estimated) the marginal distributions
- We know (or have estimated) the copula that describes the dependencies
- We can simulate *n* values from each marginal
- We can apply the dependency described by the copula through a reordering algorithm

We have *n* scenarios for the Total P&L, with all the information about the marginals that created those scenarios

Standalone Approach in practice

TOTAL		CREDIT		PREM NONCAT		PREM NATCAT	
Base Scenario Total		Base Scenario		Base Scenario		Base Scenario	
MC_1	- 383,383	MC_1	- 2,349	MC_1	- 284,997	MC_1	- 24,394
MC_2	66,649	MC_2	4,539	MC_2	217,133	MC_2	21,487
MC_3	444,444	MC_3	7,491	MC_3	66,689	MC_3	5,481
MC_4	407,828	MC_4	6,507	MC_4	- 173,492	MC_4	11,180
MC_5	185,757	MC_5	- 381	MC_5	- 73,747	MC_5	- 23,569
MC_6	- 573,607	MC_6	- 13,173	MC_6	147,894	MC_6	- 48,744
MC_7	93,736	MC_7	3,555	MC_7	86,124	MC_7	13,541
MC_8	- 146,126	MC_8	- 3,333	MC_8	82,130	MC_8	- 30,636
MC_9	-1,770,874	MC_9	- 82,055	MC_9	- 330,367	MC_9	- 12,234
MC_10	- 291,491	MC_10	4,539	MC_10	- 390,677	MC_10	17,539
MC_11	- 371,803	MC_11	- 13,173	MC_11	- 20,013	MC_11	- 30,749
MC_12	135,088	MC_12	1,587	MC_12	304,842	MC_12	- 6,347
MC_13	446,603	MC_13	4,539	MC_13	160,312	MC_13	- 16,983
MC_14	790,223	MC_14	9,459	MC_14	348,323	MC_14	15,415
MC_15	652,424	MC_15	7,491	MC_15	175,594	MC_15	- 32,936
MC_16	1,194,650	MC_16	9,459	MC_16	196,472	MC_16	- 19,309
MC_17	- 620,535	MC_17	- 12,189	MC_17	- 103,331	MC_17	- 578
MC_18	206,989	MC_18	5,523	MC_18	172,769	MC_18	7,663
MC_19	- 352,858	MC_19	7,491	MC_19	- 201,352	MC_19	18,581
MC_20	421,706	MC_20	1,587	MC_20	146,941	MC_20	- 9,109
MC_21	94,541	MC_21	7,491	MC_21	21,552	MC_21	11,711
MC_22	- 24,103	MC_22	- 4,317	MC_22	213,096	MC_22	- 53,112
MC_23	- 289,611	MC_23	- 9,237	MC_23	128,032	MC_23	- 12,162
MC_24	116,418	MC_24	- 381	MC_24	255,703	MC_24	- 21,580
MC_25	516,682	MC_25	6,507	MC_25	139,504	MC_25	24,488
MC_26	- 769,784	MC_26	1,587	MC_26	- 179,759	MC_26	15,845
MC_27	269,004	MC_27	4,539	MC_27	- 243,404	MC_27	14,115
MC_28	760,991	MC_28	9,459	MC_28	86,041	MC_28	11,891
MC_29	10,713	MC_29	603	MC_29	- 234,246	MC_29	- 18,798
MC_30	- 554,817	MC_30	- 381	MC_30	- 106,315	MC_30	15,249
MC_31	-1,232,559	MC_31	- 65,327	MC_31	- 213,586	MC_31	- 26,133
MC_32	- 142,095	MC_32	1,587	MC_32	- 253,176	MC_32	- 30,569
MC_33	796,201	MC_33	10,443	MC_33	329,991	MC_33	25,961
MC_34	- 96,648	MC_34	1,587	MC_34	- 213,490	MC_34	11,117
MC_35	- 17,751	MC_35	3,555	MC_35	84,257	MC_35	- 6,160
MC_36	135,934	MC_36	4,539	MC_36	99,604	MC_36	4,951
MC_37	670,321	MC_37	9,459	MC_37	195,116	MC_37	15,487
MC_38	- 420,353	MC_38	4,539	MC_38	- 15,992	MC_38	- 14,597
MC_39	357,940	MC_39	7,491	MC_39	49,986	MC_39	22,547
MC_40	- 230,699	MC_40	3,555	MC_40	190,233	MC_40	- 7,398
MC_41	924,739	MC_41	4,539	MC_41	349,147	MC_41	7,453

Choose a default risk measure ρ

Calculate the standalone RC for each risk and for the total

$RC_i = \rho(X_i)$ *for each i*

RC_{TOT} as sum of RC_i

$$\Phi_i = RC_i \, / \, RC_{TOT}$$

Reallocation of Risk Capital in practice

TOTAL		CREDIT		PREM NONCAT		PREM NATCAT	
Base Scenario Total		Base Scenario		Base Scenario		Base Scenario	
MC_1	- 389,383	MC_1	- 2,349	MC_1	- 284,997	MC_1	- 24,394
MC_2	66,649	MC_2	4,539	MC_2	217,133	MC_2	21,487
MC_3	444,444	MC_3	7,491	MC_3	66,689	MC_3	5,481
MC_4	407,828	MC_4	6,507	MC_4	- 173,492	MC_4	11,180
MC_5	185,757	MC_5	- 381	MC_5	- 73,747	MC_5	- 23,569
MC_6	- 573,607	MC_6	- 13,173	MC_6	147,894	MC_6	- 48,744
MC_7	93,736	MC_7	3,555	MC_7	86,124	MC_7	13,541
MC_8	- 146,126	MC_8	- 3,333	MC_8	82,130	MC_8	- 30,636
MC_9	-1,770,874	MC_9	- 82,055	MC_9	- 330,367	MC_9	- 12,234
MC_10	- 291,491	MC_10	4,539	MC_10	- 390,677	MC_10	17,539
MC_11	- 371,803	MC_11	- 13,173	MC_11	- 20,013	MC_11	- 30,749
MC_12	135,088	MC_12	1,587	MC_12	304,842	MC_12	- 6,347
MC_13	446,603	MC_13	4,539	MC_13	160,312	MC_13	- 16,983
MC_14	790,223	MC_14	9,459	MC_14	348,323	MC_14	15,415
MC_15	652,424	MC_15	7,491	MC_15	175,594	MC_15	- 32,936
MC_16	1,194,650	MC_16	9,459	MC_16	196,472	MC_16	- 19,309
MC_17	- 620,535	MC_17	- 12,189	MC_17	- 103,331	MC_17	- 578
MC_18	206,989	MC_18	5,523	MC_18	172,769	MC_18	7,663
MC_19	- 352,858	MC_19	7,491	MC_19	- 201,352	MC_19	18,581
MC_20	421,706	MC_20	1,587	MC_20	146,941	MC_20	- 9,109
MC_21	94,541	MC_21	7,491	MC_21	21,552	MC_21	11,711
MC_22	- 24,103	MC_22	- 4,317	MC_22	213,096	MC_22	- 53,112
MC_23	- 289,611	MC_23	- 9,237	MC_23	128,032	MC_23	- 12,162
MC_24	116,418	MC_24	381	MC_24	255,703	MC_24	- 21,580
MC_25	516,682	MC_25	6,507	MC_25	139,504	MC_25	24,488
MC_26	- 769,784	MC_26	1,587	MC_26	- 179,759	MC_26	15,845
MC_27	269,004	MC_27	4,539	MC_27	- 243,404	MC_27	14,115
MC_28	760,991	MC_28	9,459	MC_28	86,041	MC_28	11,891
MC_29	10,713	MC_29	603	MC_29	- 234,246	MC_29	- 18,798
MC_30	- 554,817	MC_30	- 381	MC_30	- 106,315	MC_30	15,249
MC_31	-1,232,559	MC_31	- 65,327	MC_31	- 213,586	MC_31	- 26,133
MC_32	- 142,095	MC_32	1,587	MC_32	- 253,176	MC_32	- 30,569
MC_33	796,201	MC_33	10,443	MC_33	329,991	MC_33	25,961
MC_34	- 96,648	MC_34	1,587	MC_34	- 213,490		
MC_35	- 17,751	MC_35	3,555	MC_35	84,257		
MC_36	135,934	MC_36	4,539	MC_36	99,604		
MC_37	670,321	MC_37	9,459	MC_37	195,116		
MC_38	- 420,353	MC_38	4,539	MC_38	- 15,992		
MC_39	357,940	MC_39	7,491	MC_39	49,986		
MC_40	- 230,699	MC_40	3,555	MC_40	190,233		
MC_41	924,739	MC_41	4,539	MC_41	349,147		

TOTAL		CREDIT		PREM NONCAT		PREM NATCAT	
Base Scenario Total		Base Scenario		Base Scenario		Base Scenario	
MC_9	-1,770,874	MC_9	- 82,055	MC_9	- 330,367	MC_9	- 12,234
MC_31	-1,232,559	MC_31	- 65,327	MC_31	- 213,586	MC_31	- 26,133
MC_42	- 780,867	MC_42	- 31,870	MC_42	109,577	MC_42	- 80,510
MC_26	- 769,784	MC_26	1,587	MC_26	- 179,759	MC_26	15,845
MC_17	- 620,535	MC_17	- 12,189	MC_17	- 103,331	MC_17	- 578
MC_6	- 573,607	MC_6	- 13,173	MC_6	147,894	MC_6	- 48,744
MC_30	- 554,817	MC_30	- 381	MC_30	- 106,315	MC_30	15,249
MC_38	- 420,353	MC_38	4,539	MC_38	- 15,992	MC_38	- 14,597
MC_1	- 389,383	MC_1	- 2,349	MC_1	- 284,997	MC_1	- 24,394
MC_11	- 371,803	MC_11	- 13,173	MC_11	- 20,013	MC_11	- 30,749
MC_19	- 352,858	MC_19	7,491	MC_19	- 201,352	MC_19	18,581
MC_10	- 291,491	MC_10	4,539	MC_10	- 390,677	MC_10	17,539
MC_23	- 289,611	MC_23	- 9,237	MC_23	128,032	MC_23	- 12,162
MC_40	- 230,699	MC_40	3,555	MC_40	190,233	MC_40	- 7,398
MC_8	- 146,126	MC_8	- 3,333	MC_8	82,130	MC_8	- 30,636
MC_32	- 142,095	MC_32	1,587	MC_32	- 253,176	MC_32	- 30,569
MC_34	- 96,648	MC_34	1,587	MC_34	- 213,490	MC_34	11,117
MC_44	- 58,980	MC_44	2,571	MC_44	126,183	MC_44	- 12,032
MC_22	- 24,103	MC_22	- 4,317	MC_22	213,096	MC_22	- 53,112
MC_35	- 17,751	MC_35	3,555	MC_35	84,257	MC_35	- 6,160
MC_29	10,713	MC_29	603	MC_29	- 234,246	MC_29	- 18,798
MC_2	66,649	MC_2	4,539	MC_2	217,133	MC_2	21,487
MC_7	93,736	MC_7	3,555	MC_7	86,124	MC_7	13,541
MC_21	94,541	MC_21	7,491	MC_21	21,552	MC_21	11,711
MC_24	116,418	MC_24	- 381	MC_24	255,703	MC_24	- 21,580
MC_12	135,088	MC_12	1,587	MC_12	304,842	MC_12	- 6,347
MC_36	135,934	MC_36	4,539	MC_36	99,604	MC_36	4,951
MC_5	185,757	MC_5	- 381	MC_5	- 73,747	MC_5	- 23,569
MC_18	206,989	MC_18	5,523	MC_18	172,769	MC_18	7,663
MC_27	269,004	MC_27	4,539	MC_27	- 243,404	MC_27	14,115
MC_43	347,359	MC_43	8,475	MC_43	- 9,981	MC_43	517
MC_39	357,940	MC_39	7,491	MC_39	49,986	MC_39	22,547
MC_45	376,026	MC_45	7,491	MC_45	- 6,839	MC_45	- 6,385
MC_4	407,828	MC_4	6,507	MC_4	- 173,492	MC_4	11,180
MC_20	421,706	MC_20	1,587	MC_20	146,941	MC_20	- 9,109
		3	7,491	MC_3	66,689	MC_3	5,481
		13	4,539	MC_13	160,312	MC_13	- 16,983
		25	6,507	MC_25	139,504	MC_25	24,488
		15	7,491	MC_15	175,594	MC_15	- 32,936
		37	9,459	MC_37	195,116	MC_37	15,487
		28	9,459	MC_28	86,041	MC_28	11,891

First step: Reorder TOTAL scenarios in order to find worst ones

Covariance Approach in practice

$$\frac{\mathrm{var}(X)}{\mathrm{var}(X)} = \frac{\mathrm{cov}(X, X)}{\mathrm{var}(X)} =$$

$$= \frac{\mathrm{cov}\left(\sum_i X_i, \sum_i X_i\right)}{\mathrm{var}\left(\sum_i X_i\right)} =$$

$$= \frac{\sum_i \mathrm{cov}\left(X_i, \sum_i X_i\right)}{\mathrm{var}\left(\sum_i X_i\right)}$$

$\Phi_i = \mathrm{cov}(X_i, X) / \mathrm{var}(X)$

var(X)

TOTAL		CREDIT		PREM NONCAT		PREM NATCAT	
Base Scenario Total		Base Scenario		Base Scenario		Base Scenario	
MC_9	-1,770,874	MC_9	- 82,055	MC_9	- 330,367	MC_9	- 12,234
MC_31	-1,232,559	MC_31	- 65,327	MC_31	- 213,586	MC_31	- 26,133
MC_42	- 780,867	MC_42	- 31,870	MC_42	109,577	MC_42	- 80,510
MC_26	- 769,784	MC_26	1,587	MC_26	- 179,759	MC_26	15,845
MC_17	- 620,535	MC_17	- 12,189	MC_17	- 103,331	MC_17	- 578
MC_6	- 573,607	MC_6	- 13,178	MC_6	147,894	MC_6	- 48,744
MC_30	- 554,817	MC_30	- 381	MC_30	- 106,315	MC_30	15,249
MC_38	- 420,353	MC_38	4,539	MC_38	- 15,992	MC_38	- 14,597
MC_1	- 389,383	MC_1	- 2,349	MC_1	- 284,997	MC_1	- 24,394
MC_11	- 371,803	MC_11	- 13,178	MC_11	- 20,013	MC_11	- 30,749
MC_19	- 352,858	MC_19	7,491	MC_19	- 201,352	MC_19	18,581
MC_10	- 291,491	MC_10	4,539	MC_10	- 390,677	MC_10	17,539
MC_23	- 289,611	MC_23	- 9,237	MC_23	128,032	MC_23	- 12,162
MC_40	- 230,699	MC_40	3,555	MC_40	190,233	MC_40	- 7,398
MC_8	- 146,126	MC_8	- 3,338	MC_8	82,130	MC_8	- 30,636
MC_32	- 142,095	MC_32	1,587	MC_32	- 253,176	MC_32	- 30,569
MC_34	- 96,648	MC_34	1,587	MC_34	- 213,490	MC_34	11,117
MC_44	- 58,980	MC_44	2,571	MC_44	126,183	MC_44	- 12,032
MC_22	- 24,103	MC_22	- 4,317	MC_22	213,096	MC_22	- 53,112
MC_35	- 17,751	MC_35	3,555	MC_35	84,257	MC_35	- 6,160
MC_29	- 10,713	MC_29	603	MC_29	- 234,246	MC_29	- 18,798

cov(X_i,X)

Decomp VaR Approach in practice

TOTAL			CREDIT			PREM NONCAT			PREM NATCAT		
Base Scenario Total			Base Scenario			Base Scenario			Base Scenario		
MC_9		-1,770,874	MC_9	-	82,055	MC_9	-	330,367	MC_9	-	12,234
MC_31		-1,232,559	MC_31	-	65,327	MC_31	-	213,586	MC_31	-	26,133
MC_42	-	780,867	MC_42	-	31,870	MC_42		109,577	MC_42	-	80,510
MC_26	-	769,784	MC_26		1,587	MC_26	-	179,759	MC_26		15,845
MC_17	-	620,535	MC_17	-	12,189	MC_17	-	103,331	MC_17	-	578
MC_6	-	573,607	MC_6	-	13,173	MC_6		147,894	MC_6	-	48,744
MC_30	-	554,817	MC_30	-	381	MC_30	-	106,315	MC_30		15,249
MC_38	-	420,353	MC_38		4,539	MC_38	-	15,992	MC_38	-	14,597
MC_1	-	389,383	MC_1	-	2,349	MC_1	-	284,997	MC_1	-	24,394
MC_11	-	371,803	MC_11	-	13,173	MC_11	-	20,013	MC_11	-	30,749
MC_19	-	352,858	MC_19		7,491	MC_19	-	201,352	MC_19		18,581
MC_10	-	291,491	MC_10		4,539	MC_10	-	390,677	MC_10		17,539
MC_23	-	289,611	MC_23	-	9,237	MC_23		128,032	MC_23	-	12,162
MC_40	-	238,693	MC_40		3,555	MC_40		198,233	MC_40	-	7,398
MC_8	-	146,126	MC_8	-	3,333	MC_8		82,130	MC_8	-	30,636
MC_32	-	142,095	MC_32		1,587	MC_32	-	253,176	MC_32	-	30,569
MC_34	-	96,648	MC_34		1,587	MC_34	-	213,490	MC_34		11,117
MC_44	-	58,980	MC_44		2,571	MC_44		126,183	MC_44	-	12,032
MC_22	-	24,103	MC_22	-	4,317	MC_22		213,096	MC_22	-	53,112
MC_35	-	17,751	MC_35		3,555	MC_35		84,257	MC_35	-	6,160
MC_29		10,713	MC_29		603	MC_29	-	234,246	MC_29	-	18,798

Take the n^{th} scenario, corresponding to the selected alpha percentile

VaR(X)

DecompVaR(X_i)

$$\Phi_i = DecompVaR(X_i) \,/\, VaR(X)$$

VaR HD Approach in practice

Basically, the VaR_HD is an estimator of the VaR, which consist in a weighted average using the “HD weights”

Φ_i = $VaR\_HD(X_i)$ / $VaR\_HD(X)$

TOTAL		CREDIT		PREM NONCAT		PREM NATCAT	
Base Scenario Total		Base Scenario		Base Scenario		Base Scenario	
MC_9	-1,770,874	MC_9	- 82,055	MC_9	- 330,367	MC_9	- 12,234
MC_31	-1,232,559	MC_31	- 65,327	MC_31	- 213,586	MC_31	- 26,133
MC_42	- 780,867	MC_42	- 31,870	MC_42	109,577	MC_42	- 80,510
MC_26	- 769,784	MC_26	1,587	MC_26	- 179,759	MC_26	15,845
MC_17	- 620,535	MC_17	- 12,189	MC_17	- 103,331	MC_17	- 578
MC_6	- 573,607	MC_6	- 13,173	MC_6	147,894	MC_6	- 48,744
MC_30	- 554,817	MC_30	- 381	MC_30	- 108,315	MC_30	15,249
MC_38	- 420,353	MC_38	4,539	MC_38	- 15,992	MC_38	- 14,597
MC_1	- 389,383	MC_1	- 2,349	MC_1	- 284,997	MC_1	- 24,394
MC_11	- 371,803	MC_11	- 13,178	MC_11	- 20,013	MC_11	- 30,749
MC_19	- 352,858	MC_19	7,491	MC_19	- 201,352	MC_19	18,581
MC_10	- 291,491	MC_10	4,539	MC_10	- 390,677	MC_10	17,539
MC_23	- 289,611	MC_23	- 9,237	MC_23	128,032	MC_23	- 12,162
MC_40	- 238,699	MC_40	3,555	MC_40	198,233	MC_40	- 7,398
MC_8	- 146,126	MC_8	- 3,338	MC_8	82,130	MC_8	- 30,636
MC_32	- 142,095	MC_32	1,587	MC_32	- 253,176	MC_32	- 30,569
MC_34	- 96,648	MC_34	1,587	MC_34	- 213,490	MC_34	11,117
MC_44	- 58,980	MC_44	2,571	MC_44	126,183	MC_44	- 12,032
MC_22	- 24,103	MC_22	- 4,317	MC_22	213,096	MC_22	- 53,112
MC_35	- 17,751	MC_35	3,555	MC_35	84,257	MC_35	- 6,160
MC_29	10,713	MC_29	603	MC_29	- 234,246	MC_29	- 18,798

$VaR\_HD(X)$ = $E(X$ | selected scenarios around VaR(X))

$VaR\_HD(X_i)$ = $E(X_i$ | selected scenarios around VaR(X))

TVaR Approach in practice

Consider *n* (alpha-driven) scenarios

↓

Calculate expected values on selected scenarios

$$\frac{TVaR(X)}{TVaR(X)} = \frac{TVaR\left(\sum_i X_i\right)}{TVaR(X)} =$$

$$= \frac{\sum_i TVaR(X_i)}{TVaR(X)}$$

$\Phi_i = TVaR(X_i) \,/\, TVaR(X)$

TOTAL		CREDIT		PREM NONCAT		PREM NATCAT	
Base Scenario Total		Base Scenario		Base Scenario		Base Scenario	
MC_9	-1,770,874	MC_9	- 82,055	MC_9	- 330,367	MC_9	- 12,234
MC_31	-1,232,559	MC_31	- 65,327	MC_31	- 213,586	MC_31	- 26,133
MC_42	- 780,867	MC_42	- 31,870	MC_42	109,577	MC_42	- 80,510
MC_26	- 769,784	MC_26	1,587	MC_26	- 179,759	MC_26	15,845
MC_17	- 620,535	MC_17	- 12,189	MC_17	- 103,331	MC_17	- 578
MC_6	- 573,607	MC_6	- 13,178	MC_6	147,894	MC_6	- 48,744
MC_30	- 554,817	MC_30	- 381	MC_30	- 106,315	MC_30	15,249
MC_38	- 420,353	MC_38	4,539	MC_38	- 15,992	MC_38	- 14,597
MC_1	- 389,383	MC_1	- 2,349	MC_1	- 284,997	MC_1	- 24,394
MC_11	- 371,803	MC_11	- 13,178	MC_11	- 20,013	MC_11	- 30,749
MC_19	- 352,858	MC_19	7,491	MC_19	- 201,352	MC_19	18,581
MC_10	- 291,491	MC_10	4,539	MC_10	- 390,677	MC_10	17,539
MC_23	- 289,611	MC_23	- 9,237	MC_23	128,032	MC_23	- 12,162
MC_40	- 230,699	MC_40	3,555	MC_40	190,233	MC_40	- 7,398
MC_8	- 146,126	MC_8	- 3,338	MC_8	82,130	MC_8	- 30,636
MC_32	- 142,095	MC_32	1,587	MC_32	- 253,176	MC_32	- 30,569
MC_34	- 96,648	MC_34	1,587	MC_34	- 213,490	MC_34	11,117
MC_44	- 58,980	MC_44	2,571	MC_44	126,183	MC_44	- 12,032
MC_22	- 24,103	MC_22	- 4,317	MC_22	213,096	MC_22	- 53,112
MC_35	- 17,751	MC_35	3,555	MC_35	84,257	MC_35	- 6,160
MC_29	10,713	MC_29	603	MC_29	- 234,246	MC_29	- 18,798

TVaR(*X*) =
E(*X* | selected scenarios)

TVaR(X_i) =
E(X_i | selected scenarios)

Reallocation in Practice - comments

CONs of the Methodologies described	
Covariance Approach	Ok if we are restricted to a low number of sims, but we are not looking to the tails of the distribution, and therefore we could underestimate the RC reallocated to fat tailed marginals
Decomp VaR Approach	Too much volatility in the allocation, this method is not reliable
VaR HD Approach	Decent allocation method if we want to use the VaR for the total RC, but can show undesired effects (e.g. the diversifying allocation property is not always valid, especially for fat tailed distributions)
TVaR Approach	No cons? ☺

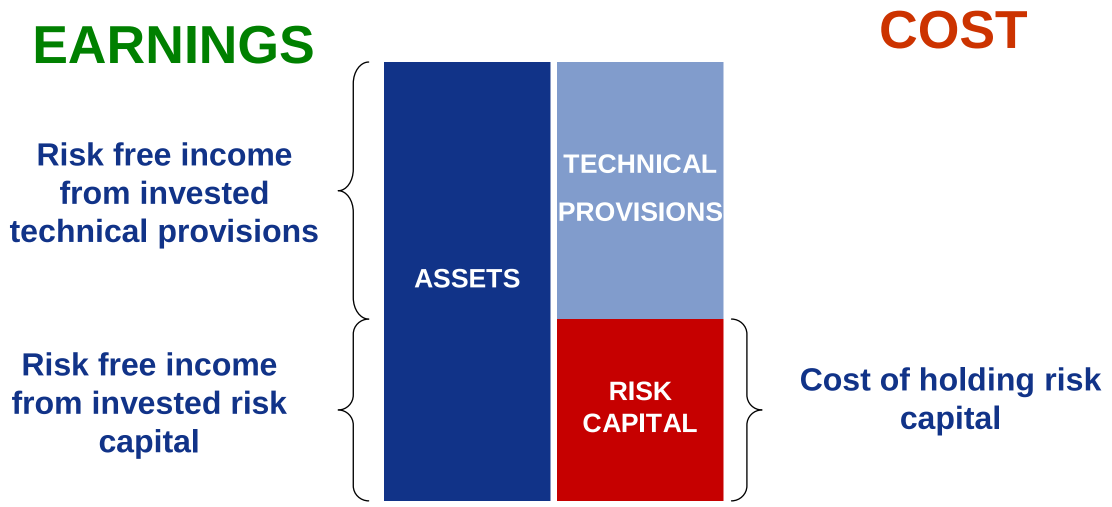
Contents

Risk Based Pricing

Consider a single P&C policy; its margin is generated by:

Technical profit = premium - loss – expenses → *1 - CoR*

…but often only the interest income is taken into account…



Opportunity cost = **RISK FREE INCOME** **- COST OF RISK CAPITAL (CoC)**

…so the idea is to rewrite the formula, taking into account also risk based opportunity cost with both interest income and cost of capital…

Risk Based Pricing

Why should be use risk based pricing?

(*) all figures are illustrative only, they don't represent actual capital charges under SII

Product A

- Observed CoR 100%
- Profit from investments 3%
- Allocated CoC 1%

Technical Profit: 0
With Financial Profit: 3%
With CoC: +2%

Product B

- Observed CoR 97%
- Profit from investments 1%
- Allocated CoC 5%

Technical Profit: 3%
With Financial Profit: 4%
With CoC: -1%

Product B, even it seems more profitable (on the technical side) than Product A, in practice lead the company to a loss…without RBP we run the risk of selling non profitable products!

(**) In this terms, for example, the Solvency 1 formula leads to a "constant" CoC of 18% * 6% (CoC) * 3 (duration) ~ 3%

Risk Based Pricing

We get an EVA (or Economic Combined Ratio) based approach:

$$EVA_{net} = \text{Prem}_{net} - \text{Losses}_{net} - \text{Expenses}_{net} \boxed{- \text{CoC}_{net} + \text{RF}_{net}}$$

Underlining the Reinsurance EVA, a risk-based measure of the profitability of the reinsurance, we get:

$$EVA_{net} = \text{Prem}_{gross} - \text{Losses}_{gross} - \text{Exp}_{gross} - \text{CoC}_{gross} + \text{RF}_{gross} + \text{Reinsurance}_{EVA}$$

Where:

$$\text{Reinsurance}_{\text{EVA}} = \text{Losses}_{ceded} + \text{Comm} + \text{CoC}_{released} - \text{Prem}_{ceded} - \text{RF}_{ceded}$$

Risk Based Pricing

If we divide the previous formula by the Gross Earned Premiums, we obtain:

$$\frac{EVA_{net}}{\mathrm{Prem}_{gross}} = 1 - \mathrm{LoR}_{gross} - \mathrm{ExR}_{gross} - \mathrm{CoC\_Ratio}_{gross} + \mathrm{RF\_Ratio}_{gross} + \mathrm{Reinsurance\_Ratio}_{EVA}$$

Defined all quantities as function of the Gross LoR and given a net profit level, our aim is to find the target Gross LoR (via goal seek) that makes the equation go to zero:

$$1 - \mathbf{LoR}_{\mathbf{gross}} - \widehat{\mathrm{profit}}_{net} - \widehat{\mathrm{ExR}}_{gross} - \boxed{\mathrm{CoC\_Ratio}_{gross} + \mathrm{RF\_Ratio}_{gross} + \mathrm{Reinsurance\_Ratio}_{EVA}} \rightarrow 0$$

Function(*LoR*)

In this way, setting for example the profit = 0, we get a “break even” CoR, that could help in business steering

Risk Based Pricing

An example of application (no reinsurance):

Product A (Property Non-Cat)

- Profit ratio set at 5%
- Rf ratio set at 2%
- CoC ratio set as 1%

Traditional View

Target CoR: 95% (97% if we consider financial result)

Economic View

Target CoR: 95% + 2% (Rf) - 1% (CoC) = 96%

Product B (Property w/Cat)

- Profit ratio set at 5%
- Rf ratio set at 2%
- CoC ratio set as 10%

Traditional View

Target CoR: 95% (97% if we consider financial result)

Economic View

Target CoR: 95% + 2% (Rf) - 10% (CoC) = 87%

Even if the product seem similar from a technical perspective, on an economic basis they lead to different conclusions; also Reinsurance could be used strategically!

Evaluating the Cost of Risk Capital

Taking again the EVA formula …

$$1-\mathbf{LoR}_{\mathbf{gross}}-\widehat{\text{profit}}_{net}-\widehat{\text{ExR}}_{gross}-\text{CoC\_Ratio}_{gross}+\text{RF\_Ratio}_{gross}+\text{Reinsurance\_Ratio}_{EVA}\rightarrow 0$$

- Net profit ratio is set by the pricing department
- Expense ratio is set by the pricing department, in order to match **pure premium** with the assumptions on losses used by the pricing models
- RF ratio quite easy to estimate, based on cash-flow projection of losses and curve rate assumption (that are given inputs)
- Reinsurance EVA ratio it's similar (as logic) to the EVA for the insurer and we link it to RIO results / outputs (another presentation on the topic? ☺)
- At the moment, we have not explicitly considered the CoC released by the reinsurance (embedded in the CoC net)

In the next section we will focus on how we derive directly the **net** CoC Ratio

Evaluating the Cost of Risk Capital

… how to set the capital charge?

STEP 1. RC allocation per LoB	STEP 2. Pricing Estimate
▪ Done for business monitoring, quarterly reporting ▪ Volume measures are: **technical provisions** and **planned premiums** ▪ Losses (i.e. Reserves) have a “Retrospective view”	▪ Done during design of new products ▪ Volume measures are: **planned losses** (i.e. LoR) and **planned premiums** ▪ Focus on a single (future) CY ▪ **Market Risks** out of scope

Determines the RC charges per Premium and Reserves units ⟷ **RC charges should be coherent with the “Retrospective view”**

In this way, we are assuming that new products will have underlying portfolios

Evaluating the Cost of Risk Capital

STEP 1. RC allocation per LoB

We perform the following steps:

(1) Evaluation of INTRA-RISK ALLOCATION

(2) Evaluation of INTRA-LoB ALLOCATION

(3) Calculation of RC charges per PREMIUM / RESERVE UNIT

Evaluating the Cost of Risk Capital

STEP 1. RC allocation per LoB => INTRA-RISK allocation

The aim is to assign the risk allocations to each risk type. Many methods could be chosen:

1. **TVaR** allocation @99%
2. Covariance allocation
3. Proportional method
4. Etc.

Everything depends on the model resolution, i.e. on how we derive the Capital Requirements (for example the Standard Formula don't allow a TVaR allocation)

Evaluating the Cost of Risk Capital

STEP 1. RC allocation per LoB => INTRA-LoB allocation

The aim is to assign the risk allocations to each LoB.

LoB Name		MARKET + CR RISK	CAT RISK	TERROR RISK	PREMIUM RISK	RESERVE RISK	BUSINESS RISK	OP RISK
	Standalone Risk	2,774,534	286,687	144,617	4,202,680	2,998,509	438,151	531,152
	Method 1	59.22%	19.96%	12.48%	33.27%	49.90%	23.36%	89.29%
	Method 2	60.00%	60.00%	60.00%	60.00%	60.00%	60.00%	60.00%
PRODUCT 1	521,878	164,307	6,867	3,608	139,810	149,626	10,236	47,424
PRODUCT 2	263,713	82,153	8,011	-	69,905	74,813	5,118	23,712
PRODUCT 3	929,335	377,905	4,578	3,608	83,886	344,140	6,142	109,076
PRODUCT 4	1,460,153	558,643	2,861	3,608	209,715	508,729	15,354	161,243
PRODUCT 5	537,087	65,723	12,017	5,412	349,524	59,850	25,591	18,970
PRODUCT 6	524,345	98,584	7,439	-	279,619	89,776	20,472	28,455
PRODUCT 7	952,785	295,752	15,450	1,804	265,638	269,327	19,449	85,364

… and we are able to allocate the quarterly RC for monitoring / reporting purpose!!

Evaluating the Cost of Risk Capital

STEP 1. RC allocation per LoB => RC Charges per Premium and Reserve Unit

Finally we get the risk capital charges, per Premium and Reserve (gross of reinsurance) units, depending on risk types

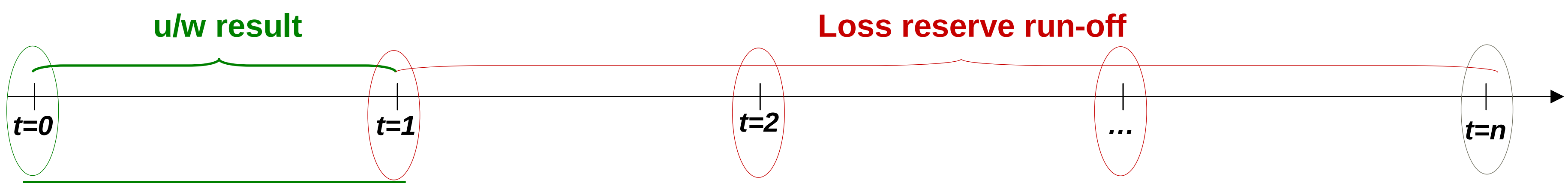
	MARKET + CR RISK	CAT RISK	TERROR RISK	PREMIUM RISK	RESERVE RISK	BUSINESS RISK	OP RISK
LoB Name	**/ Reserve**	**/ Premium**	**/ Premium**	**/ Premium**	**/ Reserve**	**/ Reserve**	**/ Reserve**
PRODUCT 1	39%	0.92%	0.48%	18.63%	35.27%	2.41%	11.18%
PRODUCT 2	43%	2.18%	0.00%	18.99%	39.35%	2.69%	12.47%
PRODUCT 3	232%	0.58%	0.45%	10.58%	211.68%	3.78%	67.09%
PRODUCT 4	54%	0.27%	0.34%	19.47%	48.89%	1.48%	15.50%
PRODUCT 5	1%	0.28%	0.13%	8.26%	0.85%	0.36%	0.27%
PRODUCT 6	2%	1.24%	0.00%	46.59%	2.25%	0.51%	0.71%
PRODUCT 7	27%	3.23%	0.38%	55.52%	24.18%	1.75%	7.66%

Given those, we need additional portfolio run-off assumptions to derive the whole Cost of Capital related to the product

Evaluating the Cost of Risk Capital

STEP 2. Pricing Estimate

- evaluation in *t*=0
- all the premiums earned only in *t=0*+ (no unearned premium reserves set)
- business runs off until the end of payments
- **market risks out of scope**



t=0
Reserve=0
NEP Plan[0,1]=P
Reserve Risk=0
Premium Risk=f(P)

t=1,…,n-1
Reserve=Rt
NEP Plan[t,t+1]=0
Reserve Risk=f(Rt)
Premium Risk=0

t=n
Reserve=0
NEP Plan[n,n+1]=0
Reserve Risk=0
Premium Risk=0

- **Premium risk arises only in *t=0***, due to the planning u/w for the period [0,1]
- **Reserve risk arises for all futures times** (*t=1* to *t=n-1*), until the full run-off of the reserve in *t=n*
- *Business and Operational Risks* rise until complete run-off

Evaluating the Cost of Risk Capital

STEP 2. Pricing Estimate

We project future reserves (using a DY pattern) → **We apply the capital charges estimated at STEP (1)**

SOLVENCY 2		*Proxy of the run-off*			
Technical Provisions	*t = 0*	*1*	*2*	*3*	*4*
BE	57.26%	23.86%	3.40%	1.05%	…
NET CoC RATIO (MVM)	**2.70%**	1.13%	0.16%	0.05%	…
LOSS TECHNICAL PROV	59.97%	24.98%	3.56%	1.10%	…
Capital Requirement					
Market + Credit Risk	24.75%	10.31%	1.47%	0.45%	…
Premium Risk Nat-Cat	2.18%	0.00%	0.00%	0.00%	…
Premium Risk Terror	0.00%	0.00%	0.00%	0.00%	…
Premium Risk Non-Cat	18.99%	0.00%	0.00%	0.00%	…
Reserve Risk	0.00%	9.39%	1.34%	0.41%	…
Business Risk	1.54%	0.64%	0.09%	0.03%	…
Operational Risk	7.14%	2.98%	0.42%	0.13%	…
	29.85%	13.01%	1.85%	0.57%	…

Evaluating the Cost of Risk Capital

PROs	
1. Immediate reconciliation with quarterly reporting	✓
2. Easy to make sensitivity analyses, especially on diversification	✓
3. Being all explicit, it's easy to communicate with other departments, in particular for risk explanation	✓

CONs	
1. Reserve Risk implicitly allows for diversification within AYs, but it makes sense if we consider a new product for an existing portfolio	☒

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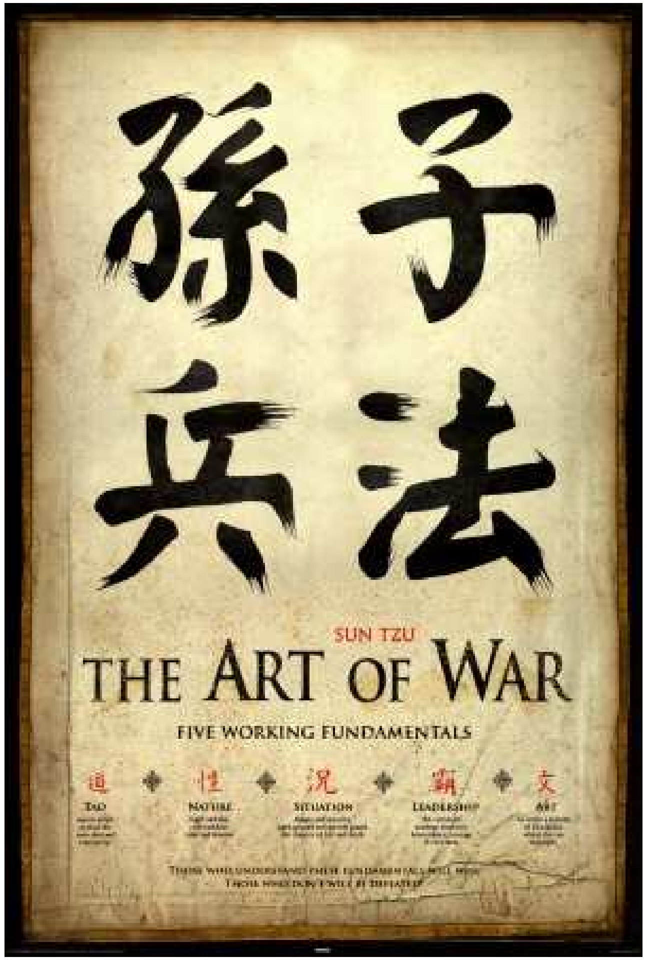

*“Ora, la forma dell’operazione militare è come quella dell’acqua. L’acqua, quando scorre, fugge le altezze e precipita verso il basso. L’operazione militare vittoriosa evita il pieno e colpisce il vuoto. Come l’acqua adegua il suo movimento al terreno, La vittoria in guerra si consegue adattandosi al nemico. L’abile condottiero non segue uno **shih** prestabilito e non mantiene una forma immutabile.*

Modificare la propria tattica adattandosi al nemico è ciò che si intende per ‘divino’*.”*

Thank you
for your attention.

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References for Risk Capital Allocation

[1] Artzner, P., F. Delbaen, J. M. Eber, and D. Heath. (1999). “Coherent Measures of Risk”. *Mathematical Finance, 9 (3), June, 1999, pp. 203–228.*

[2] Acerbi, C., Tasche, D. (2002). “On the coherence of expected shortfall”. *Journal of Banking and finance 26, pp. 1487-1503.*

[3] Bargès, M., Cossette, H. and Marceau, E. (2009). “TVaR-based capital allocation with copulas”. *Working Papers, HAL.*

[4] Denault, M. (2001). “Coherent allocation of risk capital”. *Journal of risk, 4(1):1–34.*

[5] Dhaene, J., Tsanakas, A., Valdez, E. A. and Vanduffel, S. (2011). “Optimal Capital allocation Principles”. *Journal of Risk and Insurance, 78.*

[6] Hallerbach, W.G. (2003). “Decomposing Portfolio Value-at-Risk: A General Analysis”. *The Journal of Risk 5/2, Winter, pp.1-18.*

[7] Kalkbrenner, M., H. Lotter, and L. Overbeck. (2004). “Sensible and efficient capital allocation for credit portfolios”. *Risk, 17:S19-S24.*

References for Risk Capital Allocation

[8] Overbcek, L. (2004). “Spectral Capital Allocation”. *In: Economic Capital Hrsg.: Das, A. Risk Books, London.*

[9] Tasche, D. (2000). “Risk Contributions and Performance Measurement”. *Working paper, Munich University of Technology.*

[10] Tasche, D. (2004). “Allocating portfolio economic capital to sub-portfolios”. *Economic Capital: A Practitioner Guide, pages 275-302. Risk Books.*

[11] Tasche, D. (2007). “Euler Allocation: Theory and Practice”. *Working paper.*

[12] Urban, M., J. Dittrich, C. Kluppelberg, and R. Stolting. (2004). “Allocation of risk capital to insurance portfolios”. *Blatter der DGVFM, 26:389-406.*

[13] Yamai Y., T. Yoshiba. (2007). “Comparative analyses of expected shortfall and var: their estimation error, decomposition, and optimization”. *IMES Discussion Paper No. 2001-E-12, Bank of Japan.*